Symbolic Logic

COURSE CODE: M23PH07DC

Discipline Core Course

Postgraduate Programme in Philosophy

Self Learning Material

SREENARAYANAGURU OPEN UNIVERSITY

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The State University for Education, Training and Research in Blended Format, Kerala

SREENARAYANAGURU OPEN UNIVERSITY

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To be benchmarked as a model for conservation and dissemination of knowledge and skill on blended and virtual mode in education, training and research for normal, continuing, and adult learners.

Pathway

Access and Quality define Equity.

Symbolic Logic

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Semester - II

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(With Model Question Paper Sets)

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MESSAGE FROM VICE CHANCELLOR

Dear learner,

I extend my heartfelt greetings and profound enthusiasm as I warmly welcome you to Sreenarayanaguru Open University. Established in September 2020 as a state-led endeavour to promote higher education through open and distance learning modes, our institution was shaped by the guiding principle that access and quality are the cornerstones of equity. We have firmly resolved to uphold the highest standards of education, setting the benchmark and charting the course.

The courses offered by the Sreenarayanaguru Open University aim to strike a quality balance, ensuring students are equipped for both personal growth and professional excellence. The University embraces the widely acclaimed "blended format," a practical framework that harmoniously integrates Self-Learning Materials, Classroom Counseling, and Virtual modes, fostering a dynamic and enriching experience for both learners and instructors.

The University aims to offer you an engaging and thought-provoking educational journey. The postgraduate programme in Philosophy is designed to be a continuation of the undergraduate programme in Philosophy. It maintains a close connection with the content and teaching methods of the undergraduate programme. It advances the more nuanced aspects of philosophical theories and practices. The university has recognised that empirical methods have limitations when explaining philosophical concepts. As a result, they have made a deliberate effort to use illustrative methods throughout their content delivery. The Self-Learning Material has been meticulously crafted, incorporating relevant examples to facilitate better comprehension.

Rest assured, the university's student support services will be at your disposal throughout your academic journey, readily available to address any concerns or grievances you may encounter. We encourage you to reach out to us freely regarding any matter about your academic programme. It is our sincere wish that you achieve the utmost success.

Warm regards.
Dr. Jagathy Raj V. P.

01-10-2024

Contents

BLOCK 1
Introduction

UNIT 1
Introduction

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- comprehend the definition and origin of Logic
- identify the nature, scope and concerns of Logic
- analyse the process of inference
- appreciate the role of Logic as the Science of Sciences

Background

Logic is a branch of philosophy that deals with thinking. Logic is fundamental to any thinking process; by virtue of correct thinking, we can gain the right knowledge. Hence, logic forms the foundation for all disciplines of knowledge. It deals with the correctness of thinking and investigates the relation between the elements of thought for thought to be correct. It prescribes the norms for correct thinking, and hence, it is significant for all avenues that involve thinking. This makes logic a foundational discipline, While one branch of logic, inductive logic, forms the foundation of all empirical disciplines, the other branch, deductive logic, forms the foundation of all formal knowledge.

Symbolic logic, also known as *Mathematical Logic*, is an advanced branch of logic that is more technical and precise in its application. It is used to evaluate the process of thinking by symbolising complex sentences of arguments and manipulating those symbols. Thus, symbolic logic evaluates arguments accurate and fast.

Keywords

Logic, Reflective Thinking, Inference, Premises, Conclusion, Argument, Proposition

Discussion

1.1.1 Introduction

- Etymologically, logic means 'the science of thinking or reasoning'

Logic is the branch of philosophy concerned with the use and study of valid reasoning. The word 'logic' is derived from the Greek word 'logos', which means *word*, *speech*, *reason*, or *thought*. Thus, etymologically, logic means 'the science of thinking or reasoning'. According to Charles Pierce, the central concern of logic is the classification of good and bad arguments. However, looking for a definition of logic will lead to hundreds of distinct but closely related definitions. They are all centred around classifying arguments by which the good ones are differentiated from the bad ones.

- Logic deals with reflective thinking, which is result-oriented thinking

Traditionally, logic is defined as the science which investigates the process of thinking. This definition lacks clarity since all types of thoughts are not the subject matter of logic. Logic deals with a specific kind of thought process called reflective thinking. Reflective thinking is result-oriented thinking, by which one finds the solution to a problem. In reflective thinking, one directs his thought activities towards the solution of the problem (e.g. Budget preparation). There are other types of thinking, such as non-reflective thinking, where there is no control over the thought process, and it is not directed towards any end (e.g. Daydreaming).

However, the whole process in the reflective thought process is not the subject matter of logic. Both reflective and non-reflective thoughts have also been studied by psychology. The primary concern of logic is the correctness of the reflective thought process. It is not concerned with the actual psychological operations taking place while one is thinking. Matters like the actual attempts made and trial and error methods used are not the subject matter of logic; Psychology studies them.

- Correctness of the reflective thought process

Reflective thinking or reasoning is the mental process that involves inference, which is one of the means of attaining knowledge. It is the process of drawing new knowledge on the basis of the existing ones. The existing knowledge is known as the premises, and the new knowledge that is arrived at is known as the conclusion. Logic, then, is concerned with the correctness of the inferential process. An inference is correct if its premises provide sufficient ground for the truth of its

- Inference is the process of drawing new knowledge on the basis of the existing ones.
- The existing knowledge is called the premise, and the new knowledge, the conclusion

conclusion, or else, it is incorrect. The primary concern of logic, then, is the validation of inferences. In validating, we must be able to classify inferences as correct or incorrect.

It can be seen that premises are the raw materials of inferences in producing new knowledge, and the new knowledge is known as a conclusion. For example, if a person knows that vaccination protects one from polio and also that she is vaccinated, then she can conclude that she will not get affected by polio. More formally, she can say,

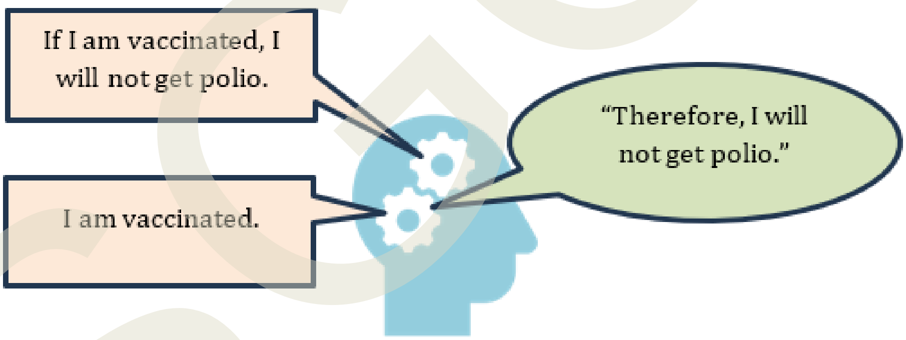
If I am vaccinated, I will not get polio.

I am vaccinated.

Therefore, I will not get polio.

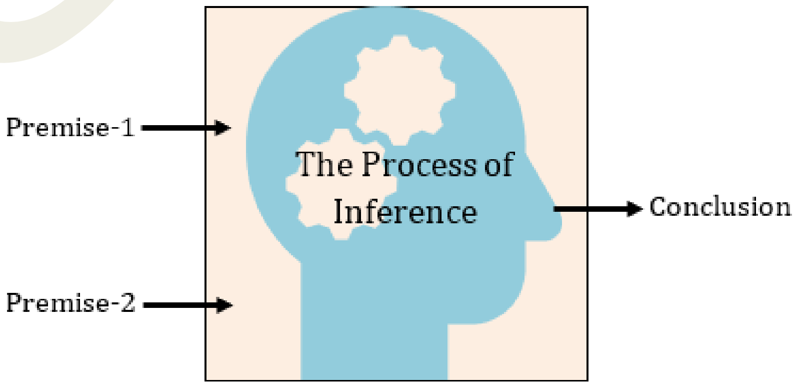
Now, the thought process in attaining the new knowledge that is going on in her mind is what is specifically called inference.

The Process of Inference



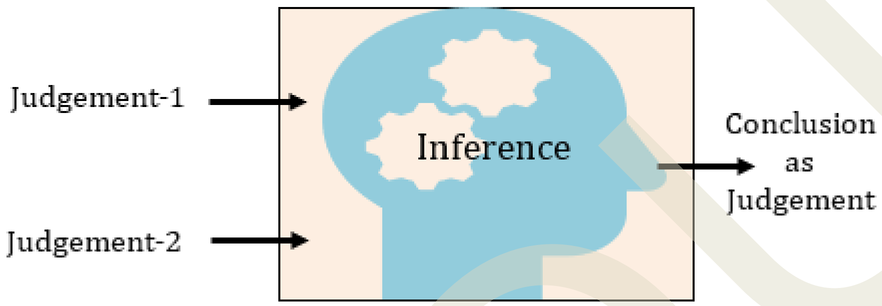
More formally,

The Process of Inference

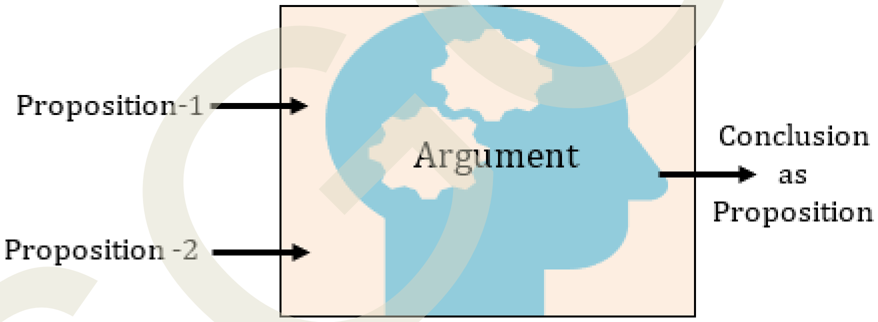


The premises or conclusions that we hold in our mind as thoughts or sentences are called judgements, while their expressions in language (when we speak them out) are called propositions. In other words, judgments in the mind, when expressed verbally, are called propositions. An inference expressed verbally is known as an argument.

When it Happens in Mind



When it is Expressed in Words



From the foregoing discussion, we may define logic as the study of methods and principles used to distinguish correct reasoning from incorrect reasoning (IM Copi).

Sophists and the Play with Logic

The Sophists were ancient Greek teachers and pre-Socratic philosophers, prominent in the 5th century BC. They were specialised in rhetoric, argumentation, and public speaking. They were known to be the first who took money for teaching. Sophists emphasised relativism, which states that truth and morality are subjective and vary depending on context. They were persuasive in their argumentation using logic. Below is a conversation between a young boy and his grandpa based on Sophism.

The room was half-open, and a cold wind was getting in. Unable to tolerate the cold, Grandpa asked the boy:

Grandpa: Aman, please close the door.

Boy: (After opening the door fully) Done it, Grandpa.

Grandpa: I told you to **close it, not to open it** (by shouting)

Boy: Grandpa, 'A door half opened is half closed'. Isn't it?

Grandpa: Yes.

Boy: Then, 'A door fully opened is fully closed'.

Grandpa: 😳

Boy: So, I've obeyed you.

Want to know such interesting argumentation?

Attempt the notion of *fallacies* in Logic.

1.1.2 Nature of Logic

Logic is a normative science: sciences may be broadly divided into two: positive and normative. Positive sciences are often studied in laboratories. Their primary concern is to explain or describe facts, events and objects within that discipline. Examples of positive sciences are zoology, physics and psychology. Normative sciences, on the other hand, explain

- Logic does not describe how things are but prescribes how they should be

how things ought to be. They are prescriptive, while positive sciences are descriptive. Normative sciences deal with the ideal ones, while positive sciences deal with the actual existents. Logic is a normative science since it prescribes the standards for correct thoughts. Other normative disciplines are ethics and aesthetics.

- In induction, a general principle is formed from particular facts
- In deduction, a particular fact is asserted based on general principles

Logical operations are either inductive or deductive: the development of logic may be traced through two distinct but closely related schemes. Traditional or Aristotelian logic, which is also called formal logic or deductive logic, is the first one. The second one is inductive logic or material logic. While inductive logic is involved in formulating general principles, deductive logic is intended to infer particular instances from these general principles. In this sense, they are distinct. They are closely related since induction supplies general principles for deductive reasoning, and deduction draws a particular conclusion for the verification of induction. One of the major differences between deduction and induction is that in deduction, the conclusion is arrived at with certainty, while in induction, the conclusion is arrived at with probability.

- Thought is a necessary aspect of the development of any science, and Logic prescribes the right method of thought

Logic is the science of sciences. Logic prescribes the right method of thought. Thought is a necessary aspect of the development of any science. The scientist himself is not involved in examining the thought process. All sciences function around formulating general principles and deducing conclusions from them. In short, the basic functions of logic- induction and deduction- are necessary for the development of any science. These principles are studied by logic, as logic is the science of sciences.

1.1.3 A Brief History of Logic

Logic plays a pivotal role in domains like philosophy, mathematics, and computer science. Like philosophy and mathematics, logic also has ancient origins. The great Greek philosopher Aristotle (384–322 BCE) is known as the father of traditional logic. His famous work, *The Organon* (*The Instrument*), is marked as the first proper and systematic study of logic. Aristotle considered logic a preparatory science. The essential elements of Aristotelean logic are categories and terms. Aristotle demonstrated the nature of the four categorical proposition and their relations. Aristotle regarded the position of terms in an argument as determining the correctness and incorrectness of an argument. In addition to this, Aristotle explains four categorical propositions, the relation of opposition between them, and a sophisticated explanation of categorical syllogism. We already noted that Aristotelean logic deals with classes. For example, the following argument comprises three classes (birds, animals, and crows.)

- Essential elements of Aristotelean logic are categories and terms

All birds are animals.

A crow is a bird.

Therefore, crow is an animal.

Chrysippus in ancient Greece, Peter Abelard, and William of Ockham are some prominent logicians. Chrysippus analysed and categorised different forms of syllogism and materialised propositional logic and asserted that the propositions are capable of being denied or affirmed. Instead of concentrating on the terms of the proposition, he focused on the whole proposition. He laid the foundations for the truth-functional interpretation of compound statements. He reasoned that the truth or falsity of compound propositions is derived from the truth or falsity of their constituent propositions.

- Propositions are capable of being denied or affirmed

After Aristotle, Medieval thinkers conceived and adopted the fundamental structure of Aristotelian logic. That means, for many centuries, the study of logic has mainly concentrated on different interpretations of the works of Aristotle. Peter Abelard (1079–1142) was the first significant logician of the Middle Ages, and he developed the logic of Aristotle and Chrysippus. Abelard identified the peculiar type of relation between premises and conclusions in deductive arguments. He differentiated the formal validity and material validity of statements and defended the priority of the formal validity of the argument. He argued that the semantic content of propositions has nothing to do with the validity of an argument. The formal relations among constituent propositions have a role in determining the validity.

- Semantic content of propositions has nothing to do with the validity of an argument

Logic is the study of arguments and reasoning. Language is the medium through which we articulate our ideas. All the arguments, thoughts, and wishes are communicated through language. Arguments are often difficult to comprehend because of the vague or unclear nature of the words used. Thus, proper communication of propositions and arguments requires the

- Logic deals with emotively neutral language

correct use of language. There are three main overlapping functions of language: informative function, expressive function, and directive function. Logic deals with the informative function of language, and reasoning is founded on factual elements of thought and abstains from the emotional aspects of thoughts. That means logical language is emotively neutral language.

- Unique vocabularies of symbolic logic are free from ambiguity

The Clear and correct use of language is necessary to formulate a valid argument. That means the vague and incorrect use of language leads to fallacies or mistakes in reasoning. To avoid difficulties related to natural language, every advanced science has developed a unique vocabulary, such as exponent symbols in mathematics and graphic formulas in organic chemistry. After Newton discovered differential calculus, mathematics achieved the status of fundamental science, and other branches of knowledge initiated the use of mathematical methods and techniques for their investigation. Logic also moved in this direction to avoid difficulties related to natural languages. Logicians thought that it was convenient to set up an artificial symbolic language free from all defects of natural language.

- German philosopher Leibniz is the pioneer of symbolic logic
- George Boole investigated the mathematical foundations of Logic

In the 17th century, the great German philosopher G.W. Von Leibniz (1646–1716) observed that traditional Aristotelean logic required modification, and he recognised the need for a universal formal language to formalise logical argument forms. Leibniz's vision was to construct a universal formal language of science that would resolve all philosophical disputes. The further development of symbolic logic was done by many philosophers and mathematicians, including Augustus De-Morgan (1806–1871), George Boole (1815–1864), William Stanley Jevons (1835–1882), and John Venn (1834–1923). Nevertheless, the initial contribution in this regard can be attributed to the English mathematician George Boole (1815-1864). In 1854, Boole published *An Investigation of the Laws of Thought*, in which he devised an algebraic system for discussing logic. In the book, he tried to find out the mathematical foundations of logic.

Boole's work was further advanced by Augustus De-Morgan and Charles Sanders Peirce. Toward the end of the nineteenth century, Gottlob Frege (1848–1925) laid the foundations of modern mathematical logic. In his famous work,

- Frege's attempt to prove the logical foundations of Arithmetic

- Russell & Whitehead, through Principia Mathematica, established the logical Foundations of mathematics almost successfully

Begriffsschrift, Frege introduced the theory of quantification. This work investigates how arithmetic is founded on Logic. The work, however, could not establish it successfully. Frege's work persisted into the twentieth century through the efforts of Alfred North Whitehead (1861–1947) and Bertrand Russell (1872–1970), whose work *Principia Mathematica* endeavoured to reduce the whole of pure mathematics to logic. As *Principia Mathematica* is the source of much of the symbolism, it is considered a paramount milestone in the history of symbolic logic.

It is interesting that all modern logicians are primarily mathematicians. In the beginning, logic borrowed basic binary operations and used variables from mathematics. However, thinkers like Bertrand Russell and Whitehead asserted that mathematics is actually using logical properties. For example, modern algebra and the set theory are based on logical properties. Thus, it is reasoned that logic is the foundation of mathematics.

Russell, his barber and the fate of Mathematics

There was a mythical town where none came from outside for work. In that town, all males were clean-shaven. There was only one male barber in that town. The following is the conversation that the barber had with a visitor to that town.

Barber: "I shave all and only those who do not shave themselves "

Visitor: "You are clean-shaven. Who shaves you?"

Barber: "I myself".

Visitor: You defined yourself (barber) as the one who shaves all and only those who do not shave themselves.

Now, if you shave yourself, you are not a barber, as the barber shaves only those who do not shave themselves.

And, if you do not shave yourself, being a barber, you should shave who do not shave themselves; that is yourself.

This famous paradox, known as the barber paradox, is the recreation of an unresolvable circularity that Russell found when he tried to establish that Mathematics is founded on Logic, using set theory. This task was taken up by Russell and Whitehead in their three-volume work *Principia Mathematica.* The attempt was successful to a great extent, ending up with a paradox called Russell's paradox, which goes like this:

Let *S* be the set of all sets that are not members of themselves. The question then is, "Is *S* a member of itself?" If *S* is an element of *S*, then *S* is a member of itself and should not be in *S*. If *S* is not an element of *S*, then *S* is not a member of itself and should be in *S*. This leads to no solution to the problem posed.

Russell approached this problem using his theory of types; the members of *Type A* sets can then only be the members of a set of *Type B* since sets of *Type B*, by definition, can contain only sets of *Type A*.

Later on, Kurt Gödel, an Austrian mathematician, criticised this position in the paper "On Formally Undecidable Propositions of Principia Mathematica and Related Systems" and declared that mathematical systems are either inconsistent or incomplete, questioning the credibility of Mathematics itself.

See *Gödel's Incompleteness Theorems* for more information.

1.1.4 Traditional Logic and Symbolic Logic

Symbolic logic is only a development and refinement of Aristotle's logic. In other words, traditional logic contains the germs of the development of symbolic logic. The distinction between classical and symbolic logic may be considered a difference in degree rather than kind. We could mention three features of symbolic logic.

i. The Use of Ideograms: These signs stand directly for concepts. For example, the multiplication sign (x) or the question mark (?) are ideograms. However, the written words "multiplication sign" or "question mark" are phonograms directly representing the spoken words.

- Symbolic logic is only a development and refinement of Aristotle's logic
- The use of ideograms, the deductive method and the use of variables are three significant features of symbolic logic

ii. The Deductive Method: Through deduction, we could develop an infinite number of new statements from a small number of statements by applying a limited number of rules.

iii. The Use of Variables: A variable is a symbol representing any of a given range of values. Symbolic logic is the method of representing logical expressions using symbols and variables rather than ordinary language.

We are familiar with the notion of variables through elementary algebra. Thus, if $x^2 = 4$ and x is a variable, the equation is valid for two values: +2 and -2. The use of variables in symbolic logic is much broader than this. Symbolic logic uses both variables and constants to represent logical connections. The extensive use of symbols is the speciality of symbolic logic. All these characteristics of symbolic logic are also characteristics of mathematics. The pioneers of logic were distinguished mathematicians. Modern symbolic logic became powerful because of the development of its technical language. The special symbols in modern logic enable us to acquire more clarity and accuracy in presenting arguments. The logical structure of arguments is made explicit by the use of special symbols. Symbolic expressions help us avoid vagueness and confusion of meaning.

- The special symbols of symbolic logic help us to exhibit the logical structure of arguments with greater clarity

Although principles and methods in traditional logic help us to represent and assess many of our most common patterns of reasoning, modern logicians have developed much more comprehensive and robust systems for expressing rational thought. Symbolic logic is the method of representing logical expressions using symbols and variables rather than ordinary language. It is noted that using particular symbolic language and logical notations is not the peculiarity of modern logic only. However, classical Aristotelian logic also uses symbols in limited ways. For instance, in traditional logic, S, P, and M are

- Traditional logic uses a limited number of symbols, whereas symbolic logic uses symbols extensively three variables used to represent subject, predicate, and middle term in a categorical syllogism. A, E, I, and O are symbols used to represent the four types of categorical propositions. Traditional logic only uses limited symbols, whereas special symbols are used extensively in symbolic logic.

- The internal structure of propositions and arguments is the focus of modern logic and not its content

Traditional logic is concerned with both form and matter of thought, and logicians use non-mathematical methods to determine the validity of arguments. Symbolic logic is purely formal, and modern logicians adopt decision procedures that ensure mathematical precision in analysing arguments. Traditional logic is more concerned with the relation of the subject and predicate terms of propositions and syllogisms. Symbolic logic is more concerned with propositions as a unit and propositional relations. The internal structure of propositions and arguments is the focus of modern logic.

- The difference between classical and symbolic logic is only of degree rather than kind

Despite all the above differences, modern logic is open to traditional logic. It is a much-improved form of traditional logic. The difference between classical and symbolic logic is only of degree rather than kind. What was implicit in Aristotelian logic has become explicit in modern logic. All traditional and modern logicians aim to provide methods to differentiate between correct and incorrect reasoning.

1.1.5 Advantages of Symbolisation

- The economic use of space, time and energy are peculiarity of symbolic logic

The use of symbols in logic has many advantages. Symbols represent ideas that would otherwise require long arguments expressed in natural language. Symbolisation makes long and big arguments precise and clear, helping us reduce the chances of errors in deciding their validity. It drastically reduces the space, time, and energy needed for logical operations.

- The use of symbols in logic makes logical analysis quicker and more accurate

The arguments formulated in English or any other natural language are often difficult to understand because of the fuzzy and equivocal nature of the words in which they are expressed. Moreover, the emotive nature of the language makes things more complicated. Symbolisation makes manipulating propositions much easier. Symbols make an argument's logical form explicit. Replacing language with symbols makes an argument's logical form apparent. When the logical form of an argument is clear, it is easy to determine its validity. Thus, the symbolic form of an argument makes logical analysis quicker and more accurate.

- Symbolic logic has the advantages of clarity, brevity, and accuracy over traditional logic

Symbolisation offers specific methods of testing the validity of arguments, like the truth table method and formal proof of validity. Thus, symbolic logic has the advantages of clarity, brevity, and accuracy over traditional logic. Using symbols helps eliminate ambiguity and reduce the vagueness of words used in arguments. We can conclude that symbols and logical notations in symbolic language act as powerful tools to determine the validity/ invalidity of arguments with clarity and accuracy.

Summarized Overview

Logic is a branch of philosophy that deals with the essence of valid reasoning. It is a normative science that prescribes criteria for correct reasoning. As underscored by Charles Pierce, logic is chiefly concerned with distinguishing between good and bad arguments. Unlike general thought processes, logic specifically investigates reflective thinking, a result-oriented thought aimed at solving problems. Logic encompasses both inductive and deductive operations, serving as a foundational tool in developing and verifying principles across disciplines, thereby positioning itself as the 'science of sciences.'

Self-Assessment

1. What is the etymological meaning of logic?
2. Differentiate reflective and non-reflective thinking.
3. What is reasoning?
4. Explain the proposition, premise and conclusion.
5. Give any two definitions of logic.
6. Distinguish between normative science and positive science.
7. How did traditional logic advance to the symbolic one?

Assignments

1. Explain the nature and scope of logic.
2. Distinguish between traditional logic and symbolic logic.

3. Give a brief history of logic.
4. Define symbolic logic. Explain the advantages of symbolisation.

Reference

1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
2. Copi, I. M., & Cohen, C. (1990). *Introduction to Logic* (9th ed.). Macmillan Publishing

Suggested Reading

1. Jain, Krishna (2013). *Textbook of Logic* (5th ed.). D.K Printworld Pvt. Ltd.
2. Klenk, V. (2007). *Understanding Symbolic Logic* (5th ed.). Prentice Hall. River, New Jersey

Space for Learner Engagement for Objective Questions

Learners are encouraged to develop objective questions based on the content in the paragraph as a sign of their comprehension of the content. The Learners may reflect on the recap bullets and relate their understanding with the narrative in order to frame objective questions from the given text. The University expects that 1 - 2 questions are developed for each paragraph. The space given below can be used for listing the questions.

UNIT 2

Propositional Logic and Predicate Logic

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- identify the distinction between propositional logic and predicate logic
- recognise the limitations of propositional logic.
- understand the fundamental principles of formal logic.
- apply the concepts of propositional and predicate logic in constructing and evaluating arguments.

Background

Formal logic, an essential domain of philosophy and mathematics, encompasses propositional and predicate logic. Propositional logic focuses on the truth values of whole propositions, while predicate logic deals with the internal structure of propositions, permitting a more subtle analysis. These logical systems have developed over centuries, with contributions from ancient philosophers like Aristotle and more recent thinkers such as Frege and Russell, influencing diverse disciplines, including mathematics, computer science, and philosophy.

Keywords

Propositional Logic, Predicate Logic, Laws of Thought, Truth and Validity

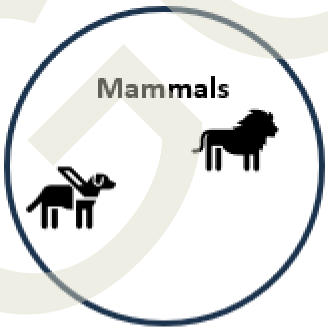
Discussion

1.2.1 Introduction

- Propositional and predicate logic are two essential components of formal logic

Propositional and predicate logic are two essential components of formal logic that furnish the framework for understanding and constructing valid arguments. Propositional logic (propositional calculus) is the branch of logic that deals with each proposition as a single unit. In propositional logic, the focus is on the truth value of propositions rather than their content. Each proposition is either true or false. A true proposition has a truth value of true (represented by the letter 'T'), while a false proposition has a truth value of false (represented by the letter 'F'). Because of this, propositional logic is often referred to as two-valued logic.

Logic is defined as the science of forms and norms. An argument consists of both form and matter. Matter refers to the meaning or content of the argument. Form relates to the arrangement, order, or pattern of this content. So, in the case of an argument, form refers to the order in which the elements of arguments (propositions of argument, including premises and conclusion) are arranged. Take the following examples,



a. Dogs are mammals.

Lions are mammals.

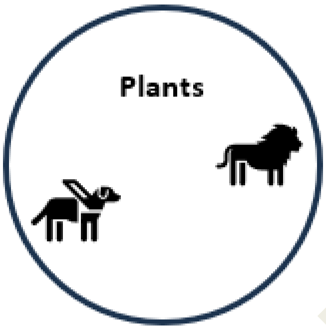
Therefore, dogs and lions are mammals.

According to the principles of logic, both the arguments (written above and below) are valid. The first one, we know, is explicitly valid as the conclusion is the combination of its premises. Here, even though both the premises are true, we need not get into their meanings; we simply need to see if the premises are true and whether the conclusion is true or not. For example, see this argument with false premises.

b. Dogs are plants.

Lions are plants.

Therefore, dogs and lions are plants.



This argument (b), too, is valid despite its premises being false. It can be seen that since both are here in the class of plants, the final statement is true if the premises are true. Hence, the argument is a valid one.

Now, consider the argument below.

c. Mammals will die

Cats are mammals.

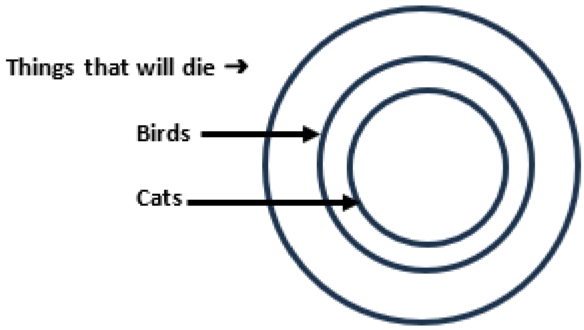
Therefore, cats will die.

Knowing that the premises are true is not sufficient here to know whether the argument is valid or not. This can be clearly understood by formulating another argument of the same form with false premises.

d. Birds will die

Cats are birds.

Therefore, cats will die.



We can see that the argument is valid using the representation above. However, this validity cannot be established merely by knowing the truth and value of the premises. Here, we need to get into the relationship between the terms within the premises. So, the propositional logic used above alone will not be sufficient here. In order to get into the inner structure of the propositions, we need the other form of logic, namely, the predicate logic.

- Validity cannot be established merely by knowing the truth of the premises

The symbolisation of propositions in propositional calculus has certain limitations. Categorical propositions possess both quantity and quality. However, in propositional calculus, there is no space for symbolising the quantity of the proposition. For example, in propositional logic, both of the following categorical propositions

All teachers are lawyers

Some teachers and lawyers are symbolised as L.

The first is universal (A proposition), and the second is particular (I proposition). However, while symbolising these propositions, the quantities of propositions are omitted. Thus, we will not be able to derive the second premise from the first one above as done in traditional logic. Take another example,

'No teachers are actors'

'Some teachers are not actors' are symbolised as ~A

The first proposition is an E proposition, and the second is an O proposition.

Thus, to address this limitation of propositional logic, a new calculus known as predicate or predicate logic is introduced. It is also called a Quantification theory. In this calculus, the proposition's internal structure is considered. Predicate logic analyses propositions into their "inner" elements, such as subjects and predicates. In predicate logic, the subject and predicate of the proposition are viewed as properties. In predicate calculus, importance is given to the predicate rather than the proposition as a whole. In the symbolisation of predicate logic, the proposition's quantity and quality are clearly expressed with equal importance.

- In propositional logic, the focus is on the form or structure of thought, whereas in predicate logic, both form and matter of thought are considered

In propositional logic, focuses on the form or structure of thought. Whereas, in predicate logic, both form and matter of thought are considered

1.2.2 Elementary Notions and Principles of Truth Functional Logic

a. The Three Laws of Thought:

- Logic is the study of the laws of thought

Traditionally, *logic* is defined as the study of laws of thought, though modern logic is not so much concerned with the laws of thought. Traditional logic considers these laws as fundamental principles that govern human thinking. The propositional logic, however, is governed by these principles.

The laws of thought are fundamental, axiomatic or self-evident rules based on rational discourse. They are viewed as self-evident because these laws cannot be proved or disproved. In order to demonstrate them, they must be accepted. To deny them is self-contradictory. They are presupposed in all rational thought and discourse. These three laws of thought are original thoughts, which cannot be reduced to another. The three laws of thought are:

- The laws of thought are fundamental axioms of logic

i. The law of identity.

ii. The law of non-contradiction.

iii. The law of the excluded middle.

The German philosopher Leibnitz added a fourth law, the law of sufficient reason, which is of little importance.

i. The Law of Identity

- The formal representation of the law of identity is 'if A is true, then it is A'

We cannot know an object if it does not have a specified character or an identical nature. It is impossible to attain knowledge of changing objects. This law says that everything is similar to itself. This law is expressed as 'whatever is, is' or symbolically as 'if anything is A, it is A.' It means that everything remains identical with itself. However, the law of identity does not ignore the fact of change. The element of identity remains through all the changes. It asserts that if any proposition is true, then it is true. By applying this law in the case of deductive argument, the meaning of terms used in the argument must remain throughout the argument. Everything in the world is changing. However, while thinking about an object, we take it as identical. If a thing is constantly altered, thought about that thing is not possible. Thus, in the realm of thought, thinking is impossible without accepting the law of identity.

ii. The Law of Non-contradiction

- A cannot be A and not A at the same time

This law does not state a new principle but only re-states the other law to make it more transparent and emphatic. We cannot say that the same man is honest and dishonest at the same time or that a student is a bachelor and married at the same time. Hence, contradictory qualities cannot be predicted for the same object simultaneously. The law of contradiction has been expressed as 'A cannot be A and not A at the same time.' In other words, a thing cannot be existent and non-existent simultaneously. The law of non-contradiction is necessary for correct thinking. It shows that coherence is the condition of human thinking.

iii. The Law of the Excluded Middle

- A must either be true or false

The law of the excluded middle says that 'A is either B or not B.' For example, when we say, 'This man is either a fool or not a fool', it is clear that if we accept one of the alternatives, we must reject the other. The acceptance of one of the alternatives means the rejection of the other. If we reject the alternative 'he is a fool,' then we must accept the other alternative, 'he is not a fool.' This law indirectly points out the fact that every affirmation carries with it a negation. These laws apply to philosophy, science, and everyday life. According to this, anything must be either true or false. The law has been expressed that anything can be either A or not A.

- Correct reasoning is not possible without assuming three laws of thought

Leibnitz added one more law. It is the Law of Sufficient Reason. It states that whatever exists or is true must have sufficient reason why it is so. This law is similar to the law of causation, which states that every event has a cause. Three laws of thought are presupposed in all rational thought and discourse. Thus, correct reasoning is not possible without assuming three laws of thought.

b. Truth and Validity

- Truth is the property of propositions, and validity is the property of deductive arguments

Truth: Truth is the property of propositions. Any proposition is either true or false. The propositions can occur either as the premise or as the conclusion of an argument.

Validity: Validity is the property of deductive arguments. Any deductive argument is either valid or invalid.

1.2.3 Relation Between Truth and Validity

- It is impossible for a valid argument to have true premises and false conclusion

There is a relation between the truth of propositions of an argument and the validity of the same. The relationship, however, is not a simple one. A deductive argument makes the claim that, since its premises are true, its conclusion is also true. In other words, the argument claims that it is valid if, whenever its premises are true, its conclusion must also be true. Hence, it is impossible for a valid argument to have true premises and false conclusion.

Hence, the only impossible relation between the truth of propositions and the validity of the argument is this: A valid argument with true premises and false conclusion. All other combinations of truth and validity are possible. That is:

a. A valid argument can have true premises and a true conclusion.

b. A valid argument can have false premises and a true conclusion.

c. A valid argument can have false premises and a false conclusion.

d. An invalid argument can have true premises and a true conclusion.

e. An invalid argument can have true premises and a false conclusion.

f. An invalid argument can have false premises and a true conclusion.

g. An invalid argument can have false premises and a false conclusion.

Arguments	Premise	Conclusion	Possible?
Valid	T	T	Y
	T	F	N
	F	T	Y
	F	F	Y
Invalid	T	T	Y
	T	F	Y
	F	T	Y
	F	F	Y

A valid argument with true premises is known as a Sound argument. The sound argument is the most useful for practical purposes.

What would have been your decision?

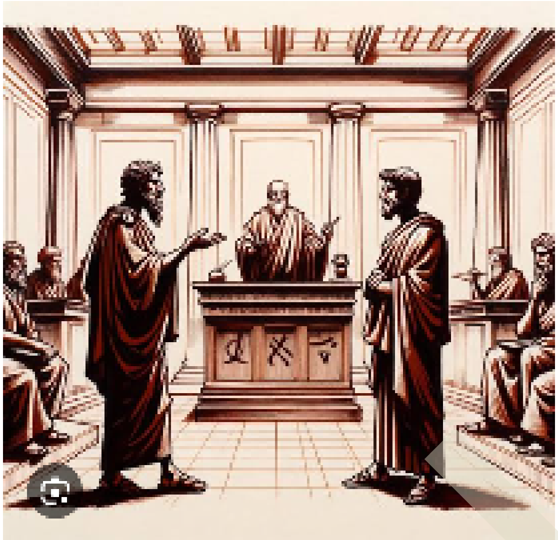
Here is a story based on an argument known as *dilemma* in Logic.

Euathlus was eager to pursue a career in law, and he sought instruction from Protagoras, a renowned sophist and expert in rhetoric. However, he lacked the funds for the tuition. To solve this, they agreed that Protagoras would train Euathlus under the condition that *payment would be made only after Euathlus won his first legal case*.

After the completion of the course, *Euathlus refrained from practising law and thus did not win any cases*. Growing impatient, Protagoras eventually sued Euathlus for the owed tuition. At last, he decided to file a case against Euathlus. When the trial began, Protagoras argued that regardless of the outcome, Euathlus must pay him. Protagoras presented his case in the form of a dilemma.

Protagoras: If Euathlus lost, he would have to pay based on the court's decision; if he won, he would have to pay according to their agreement. He must either lose or win this case, so payment is inevitable.

On the other hand, Euathlus presented his counter-argument this way:



Euathlus: If I win, I should not have to pay, as per the court's decision; if I lose, I still should not pay; as per their agreement -I would not owe anything until I win a case. Thus, I am not obliged to pay in either scenario.

If you were the judge, what would have been your decision? This is a complex scenario, even logically. Both the arguments were valid.

To know the beauty of such arguments, take a look into the ***dilemma*** *in traditional logic. The form of the argument can be studied in block 3.*

Summarized Overview

The present chapter examined the essential aspects of formal logic, focusing on propositional and predicate logic. Propositional logic deals with propositions and their truth value, emphasising the importance of form over content. Predicate logic considers both the form and matter (content) of arguments and analyses propositions' internal structure. Additionally, in this chapter, we examined the principles of truth-functional logic, outlining the traditional laws of thought and claiming that they form the basis of logical reasoning and are fundamental to understanding and constructing valid arguments in logic.

Self-Assessment

1. What is the symbolic representation of the law of non-contradiction?
2. Write the symbolic representation of the law of excluded middle.
3. What is the law of non-contradiction?
4. Explain the law of excluded middle.
5. Explain the law of identity.

Assignments

1. Explain the nature of the three laws of thought.
2. Examine the relation between truth and validity.
3. Examine the differences between propositional logic and predicate logic.

Reference

1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
2. Copi, I. M., & Cohen, C. (1990). *Introduction to Logic* (9th ed.). Macmillan Publishing

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Space for Learner Engagement for Objective Questions

Learners are encouraged to develop objective questions based on the content in the paragraph as a sign of their comprehension of the content. The Learners may reflect on the recap bullets and relate their understanding with the narrative in order to frame objective questions from the given text. The University expects that 1 - 2 questions are developed for each paragraph. The space given below can be used for listing the questions.

UNIT 3
Techniques for Symbolisation

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- translate simple statements from natural language into symbolic logic using appropriate variables and constants.
- use logical operators correctly to represent relationships between propositions.
- apply proper bracketing techniques to avoid ambiguity when writing complex logical statements.
- distinguish between variables and constants in symbolic logic, understanding their distinct roles in logical expressions.

Background

Symbolic logic focuses on translating natural language arguments into symbolic language to clarify and analyse their logical structure. It uses variables, which can represent a range of propositions, and constants, which represent fixed logical operations. This process aids in testing argument validity and developing proofs. The use of specific symbols and punctuation is necessary to sidestep the ambiguity and accurately interpret complex logical statements.

Keywords

Variables, Constant, Symbolisation, Punctuation

Discussion

1.3.1 Introduction

- Conversion of propositions and arguments from natural language to symbolic language

Elementary symbolic logic has two primary objectives. First, it focuses on converting propositions and arguments from natural language to symbolic language. Second, it aims to evaluate or test the validity of various arguments, examine the structure of statements, and develop proofs to determine if one statement logically follows from another. In this unit, we consider the initial task of symbolic logic, focusing on the essence of symbolisation and the techniques used for it. For that purpose, we use two types of symbols: variables and constants.

1.3.2 Variables and Constants

- A variable is a symbol that can stand for any one of the given range of values

There are two types of symbols in symbolic logic: variables and constants. A *variable* is a symbol that can stand for any one of the given range of values. The variable is simply a letter for which or in place of which a statement may be substituted. Compound statements, as well as simple statements, may be substituted for statement variables. To avoid confusion, we use small letters from the middle part of the alphabet, *p, q, r, s*, to *w,* as variables. A variable symbol does not have a fixed value and keeps changing from argument to argument. For example, 'p' represents "Today is Monday' in one argument and 'Students are hardworking' in another. In traditional logic, only three variable symbols, S, P, and M, were used for the various terms of a categorical syllogism.

- Constants do not change their value from proposition to proposition

Constants are symbols introduced by modern logicians. Constants do not change their value from proposition to proposition. Constants are logical operations that refer to a way of combining propositions, sentences, letters, or variables and form truth functions. Constants may be divided into monadic operators and dyadic operators. The monadic operator is related to a single proposition. Dyadic operators come in between two propositions. Negation is a monadic operator (~). The dyadic operators are conjunction (.), which stands for the relation 'and'; disjunction (v), which stands for the relation 'either-or'; implication (⊃), which stands for the relation 'if then,' and equivalence (≡), that stands for the relation 'if and only if.'

1.3.3 Basic Symbolisation Techniques

Let us learn the basics of symbolisation by symbolising the following simple propositions:

1. 'Radha is a Teacher ' is symbolised as 'T'. (Here, 'T' stands for the predicate Teacher.)

Generally, a simple proposition is symbolised using the first letter of its predicate term in uppercase. Hence,

2. 'Gita is a dancer' is symbolised as 'D.' (Here, D stands for the predicate dancer).

In the above examples, we have symbolised the proposition itself using uppercase letters. These symbols are treated as propositions themselves. In order to get into their forms, we need to represent these propositions using variables rather than constants. In symbolising so, the first simple proposition of an argument is symbolised with the lowercase letter '*p*', the next simple proposition with the lowercase letter '*q*' and so on till '*w*'.

In that way,

'Radha is a Teacher' is symbolised as ''*p*' (The first simple proposition with the letter '*p*').

'Gita is a dancer' is symbolised as '*q*'. (The second simple proposition with the letter '*q*').

It is to be noted that the sequence *p, q, r* .... needs to be followed only when these propositions are the components of one and the same argument or complex proposition.

Now, "Radha is a teacher and Geetha is a dancer" can be symbolised as:

T and D using propositions themselves.

More formally as T . D ('.' denotes 'and') and as *p . q* using propositional variables.

Consider another proposition, which is a combination of two propositions:

* In assigning variables for propositions, the first proposition is represented with the lowercase letter ‘p’, the second with the lowercase letter ‘q’ and so on till the letter ‘w’

‘Either I will go to Thalassery or attend the seminar.’

This is symbolised as ‘either G or A.’

Using variables, we can represent the above proposition as ‘Either *p* or *q*.’

Using the constant symbol “v,” we can represent it as ‘*p* v *q*.’

1.3.4 The Use of Brackets

The statements formulated in English or any other natural language are often easier to understand with punctuation marks. For example, the sentence ‘I saw a girl in the market with a telescope.’ is ambiguous without punctuation marks. It may mean;

a. There is a girl in the market, and I am watching her with my telescope.

b. There is a girl in the market who I am seeing, and she has a telescope.

c. There is a girl, and she is in the market; there is a telescope in the market.

d. I am in the market, and I see a girl using a telescope.

e. There is a girl in the market, and I am seeing her with a telescope.

* Symbolic logic requires bracketing to avoid ambiguity

Likewise, punctuation is required in the language of symbolic logic to understand complicated compound statements. Take the statement ‘p . q v r’. It might be a compound sentence whose first component is ‘p . q’, and the second component is ‘r.’ It might also refer to another compound statement whose first component is ‘p’ and the second conjunct is ‘q v r’ If we punctuate the given statement by using brackets, it will be easy to determine the type of compound proposition. We can punctuate the statement in the following two ways.

(p . q) v r

p . (q v r)

It has been remarked that symbolic logic requires punctuation to avoid ambiguity. It shares this characteristic with natural languages and other artificial languages, like (ordinary) algebra.

Why at all Punctuation? Can't We Understand from the Context?

One day, a professor wrote the following sentence on the board and told her students to punctuate it. The sentence was:

"A woman without her man is nothing."

All male students wrote like this:

"A woman, without her man, is nothing."

All the female students wrote like this:

"A woman: without her, man is nothing."

Punctuation is powerful. And, if you still think that it is not that strong and can address issues that *slightly* shift meaning like this, it is merely because you have not faced the consequences of the act of punctuation as below.

In a courtroom, the judge ordered:

"Kill him not let him live"

The clerk wrote like this:

"Kill him, not let him live."

This was what the judge meant:

"Kill him not, let him live."

As a consequence, he is not here to read this.

Summarized Overview

The symbolisation in symbolic logic is done using variables and constants representing propositions. That is, two primary symbols exist in symbolic logic: variables, which change across propositions, and constants, which are logical connectors such as "and" (.), "or" (v), "if…then" (⊃), and equivalency (≡). The use of brackets in symbolic logic plays the role of punctuation in language, provides a tool to organise statements and clarifies their intended meaning. It emphasises how punctuation and bracketing prevent ambiguity, showing multiple interpretations that can arise in natural language when punctuation is absent. More on symbolisation can be learned and applied after learning the details of logical operations like conjunction, disjunction and implication.

Self-Assessment

1. What is a variable?
2. Write a note on constant.
3. Why symbolisation is important? Explain the ambiguities in natural language and the techniques of clarifying them in symbolic logic with examples from daily life.

Assignments

1. Examine the nature of basic symbols in symbolic logic.
2. Illustrate the basic techniques of symbolisation.

Reference

1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
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BLOCK 2
Compound Propositions

UNIT 1
Simple & Compound Propositions

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- distinguish between simple and compound propositions
- identify and correctly use various types of compound statements
- understand and apply the concept of truth-functional connectives
- construct and interpret truth tables for basic logical connectives
- develop the ability to symbolise complex statements in logical form

Background

In the previous block, we saw that logic is the study of inferences, and inference is the process of obtaining new knowledge from existing knowledge. But what is 'Knowledge'?

Knowledge, generally, is held as the awareness of one or the other aspects of reality. This awareness can be of three kinds. Firstly, we are aware of people and things around us, as is the case in our familiarity with the flower rose. Secondly, we know how to carry out some activities, like driving or swimming. Thirdly, we know things by listening to others or reading; as we know, London is the capital city of the UK. The first kind of knowledge stated above is known as *knowledge by acquaintance*. The second kind is known as *procedural knowledge* or '*knowing how*'. The third type of knowledge is called descriptive *knowledge* or '*knowing that*'.

Descriptive knowledge or 'knowing that', also known as *propositional knowledge*, forms a large part of knowledge that human beings hold. This is the knowledge we acquire and retain through listening to others, reading books, etc. All this knowledge is in the form of declarative sentences or propositions. A simple proposition designates a simple fact, a complex one, and how facts are related. The importance of the role of propositions in the arena of human knowledge makes it necessary to study them in their simple and complex forms.

These declarative sentences or propositions, specifically in the context of Symbolic logic, are preferably called *statements*. This block is intended to study what simple statements are and how they make sense in their compound forms.

Keywords

Simple statement, Compound statement, Truth-functional compound statements, Conjunction, Disjunction, Negation, Implication, Biconditional

Discussion

2.1.1 Introduction

A sentence is a meaningful arrangement of words. In other words, a sentence is a group of words that conform to grammatical rules and conventions. Every meaningful sentence must be grammatically correct. E.g., “Teacher Ramu is” is a group of words without any specific meaning and is not a sentence. On the other hand, “Ramu is a teacher” is a grammatically correct sentence with a specific sense. *The specific sense expressed by a grammatically correct sentence is called a proposition*.

A sentence may have different senses and may be used in various ways. For example,

i. ‘Mother loves the child.’

ii. ‘Child is loved by its mother.’

i) and ii) are two different sentences that express the same sense and, thereby, denote the same proposition.

Take another group of sentences:

i. The sun rises in the east. (English)

ii. സൂര്യൻ കിഴക്ക് ഉദിക്കുന്നു (Malayalam)

iii. सूर्य पूर्व दिशा से उगता है । (Hindi)

iv. Sol in oriente oritur. (Latin)

The sentences given in four different languages above are different since they have different grammatical structures. However, these sentences from different languages refer to the same fact/ content, and hence they represent the same proposition.

- The sense expressed by a grammatically correct sentence is called a proposition. A proposition may either be true or false

The sense or sense expressed by a proposition may either be true or false, depending on the circumstance. Thus, the proposition "Ram killed Ravana" refers to a true proposition if and only if *it is the case that Ram killed Ravana*. The fact referred to by a proposition may either be true or false. *A proposition is said to be true if its sense conforms to reality, or else it is false*. Every proposition must either be true or false, depending on whether it refers to reality or not. A true proposition is said to have the truth value 'True' (T); a false proposition is said to have the truth value 'False' (F). Hence, every proposition has a truth value, either 'true' or 'false', and no proposition can assume a truth value between true and false. In logic, the word 'statement' is synonymous with the proposition, and the term *statement* is preferred in Symbolic Logic.

- Every sentence is not a proposition; only sentences that are declarative or factual are propositions

Sentences express thoughts, wishes, commands, questions, etc. As a result, there are different types of sentences, such as interrogative, exclamatory and declarative. Every sentence is not a proposition, but only declarative sentences express propositions. For example, the sentence "Water is essential for life" is a proposition. However, the sentence "Get me a glass of water" is not a proposition.

- Propositions are the building blocks of every argument
- The premises of an argument are the grounds for asserting the conclusion

Propositions are the building blocks of every argument. An argument is a group of propositions in which one or more of them support or provide ground for the truth of another statement. The propositions that provide the support or ground for accepting another proposition are called the premises of the argument, and the accepted proposition is called the conclusion of the argument. For example:

All flowers are beautiful.

Roses are flowers.

Therefore, roses are beautiful.   is an argument.

2.1.2 Simple and Compound Statements

- A simple statement does not contain any other statement as its components.
- A compound statement does contain simpler statements within itself

There are two types of statements: simple and compound. A simple statement is one that does not contain any other statement as its component. E.g., Roses are red. On the other hand, a compound statement does contain simpler statements as its component. More accurately, compound statements contain more than one simple statement as their component statements. For example, the statement 'Roses are red and sunflowers are yellow' is a compound statement that contains two simple statements: 'roses are red' and 'sunflowers are yellow' as its constituents. These two statements have independent sense and are connected with 'and.' Such statements, which can be divided into more than one component statement, are known as compound statements. The statements can be connected in different ways to form a compound statement. The following are various types of compound statements.

a. Conjunctive

b. Disjunctive

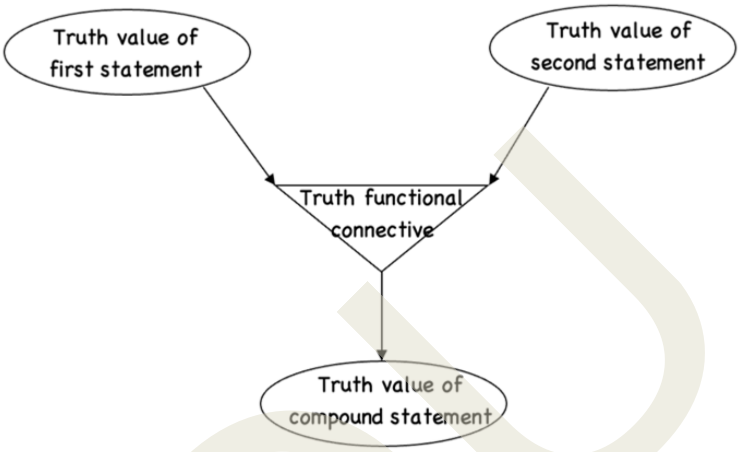
c. Implicative

d. Bi-conditional (Material Equivalence)

2.1.3 Truth Functional Compound Statements

- The truth value of a truth-functional compound statement is determined by the truth value of its component statements

A statement carries truth value. It is a description or assertion of facts. A statement is true when it describes fact correctly and false when it describes fact incorrectly. Thus, the truth value of a true statement is true, and the truth value of a false statement is false. The truth value of a compound statement is determined based on the truth values of its component statements. In such cases, the component statements that determine the truth value of a compound statement are called *truth-functional components*. We have seen that the truth values of the constituent statements determine the truth value of a truth-functional compound statement. The logical constants are the connecting links between the component statements. These constants are called **logical operators**. These logical operators, when appearing in a truth-functional compound statement, are called **truth-functional connectives**. The truth values of the component statements, along with the nature of the truth-functional connective, determine the truth value of the compound statement.



There are four types of truth-functional connectives. They are conjunction, disjunction, implication, and equivalence. Another logical operator, which is not specifically a truth-functional connective, called **negation, is a *truth modifier*** since it modifies the truth value of a single statement. Not all statements have more than one statement, as their parts are truth-functional compound statements. For example, the truth value of the statement 'Ramu believed that ghost exists' is not in any way determined by the truth value of its component simple statement 'Ghost exists.' This is so because it could be true that "Ramu believed that ghost exists", regardless of whether a ghost exists or not. So, the component "ghost exists" is not a truth-functional component of the compound statement "Ramu believed that ghost exists." Take another example: "The best friend of Alice is an IAS officer" may be split into two meaningful statements 'the best friend of Alice' and 'S/he is an IAS officer'. However, here, obviously, the partial statements are not part of the larger statement. So, the larger statement is not a compound statement and not every compound statement is truth-functional.

- Conjunction, disjunction, implication and equivalence are truth-functional connectives

It shows that two conditions must be satisfied for a complex statement to be compound.

i. The parts must be statements in their own right.

- Two conditions that make a complex statement to be compound

ii. If any part is replaced in the larger statement by any other meaningful statement, the result of that replacement must also be meaningful.

Condition ii) is not satisfied in the examples shown. For example, if we replace 'Alice's friend is an IAS officer' with 'if you work hard, then you earn money,' the resulting statement is meaningless. Thus, it is essential for a truth-functional compound statement to fulfil the above two conditions.

a) Conjunction:

- A conjunction is a compound proposition that combines two statements by inserting the word 'and' between them

A conjunction is a compound proposition that combines two statements by inserting the word 'and' between them. For example, 'Raju is tall and handsome'. The component statements in conjunction are called conjuncts. In this example, 'Raju is tall' and 'Raju is handsome' are the conjuncts of the compound statement. The truth values of conjuncts determine the truth value of the conjunction. More accurately, a conjunction is true if both conjuncts are true, or else it is false. Thus, a conjunction is a truth-functional compound statement because a conjunction's truth value is dependent upon its conjuncts' truth value.

- '.' (dot) is the symbol of conjunction

In Symbolic Logic, the symbol '.' (dot) is used to combine two statements conjunctively. The dot symbol connects the truth value of its conjuncts and is called a truth-functional connective. Using an abbreviation, the proposition 'Ramu is a teacher, and Anoop is a student' is written as T . S . Here, T and S stand for the predicate terms. Using variables (p and q), we can express this proposition as 'p. q'. In some conjunctions, both of its conjuncts have the same predicate terms. In such cases, we can use the capital letters of the subject terms to symbolise it. For example, 'Ramu and Mahesh are teachers', which can be symbolised by using the capital letters of their predicate (R. M).

Truth Table for Conjunction

Given any two statements, p and q, they can have just four possible sets of truth values. The four possible cases can be exhibited as follows:

In case p is true and q is true, p . q is true;

In case p is true and q is false, p . q is false;

In case p is false and q is true, p . q is false;

In case p is false and q is false, p . q is false.

The capital letters T and 'F' represent the truth values of true and false, respectively. This may be tabulated using T for true and F for false by using the truth table.

p	q	p . q
T	T	T
T	F	F
F	T	F
F	F	F

Truth table for Conjunction

- A conjunction is true when all of its conjuncts are true. Otherwise, it is false

The truth table defines the 'dot' symbol by showing the truth values for p . q in every possible case. It shows that a conjunction is true if and only if both statements are true. It should be noted that the words 'but, yet, also, still, although, however, nevertheless, and so on, and comma, semicolon, etc., can also be used to conjoin two statements, and they can all be represented by the 'dot' symbol.

b) Disjunction:

- When two statements are combined using the word 'or' /'either…. or….', it is called disjunction

When two statements are combined using 'either..or', the resulting compound statement is a disjunction. The word 'or' can also be used instead of 'either... or'. Disjunction is also called 'alternation'. The two statements being combined in a disjunctive statement are called the *disjuncts* or *alternatives*. The symbol used for disjunction is 'v' (wedge). There are two senses in which the disjunction is used. Example: Either Sitha is a singer or a dancer. is a disjunctive statement. For this statement to be true, Sitha must either be a singer, or she must be a dancer. However, it is possible that Sita is both a singer and a dancer. Take another example:

Sitha is either in Kochi or Chennai.

This statement, too, is true if Sitha is either in Kochi or Chennai. But, in this case, she cannot be at both places together.

So, taking these variations in the meaning of disjunctive statements, we need to account for two types of disjunctions: The *exclusive (strong) disjunction* and the *inclusive (weak) disjunction*.

i. The Exclusive (Strong) Disjunction:

- The exclusive sense of "or" is "Either A or B (but not both)'

The exclusive sense of "or" is "Either A or B (but not both). "For example, "You may go to the left or the right." In this case, both disjuncts cannot be true at the same time. You either have to turn left or right. The Latin word 'aut' expresses the exclusive or strong sense. If p and q are two statements, their strong disjunction is written as 'p ʌ q'. For example, Today is either Monday or Tuesday (but not both), which is a strong disjunction symbolised as M ʌ T.

Truth Table for Strong Disjunction:

- Four possible sets of truth values in strong disjunction

Given any two statements, p and q, they can have just four possible sets of truth values. The four possible cases of exclusive/ strong disjunction can be exhibited as follows:

In case p is true and q is true, p ^ q is false;

In case p is true and q is false, p ^ q is true;

In case p is false and q is true, p ^ q is true;

In case p is false and q is false, p ^ q is false.

A strong or exclusive disjunction is false in two cases:

i. both of its disjuncts are true,
ii. and both of its disjuncts are false

Truth Table for strong disjunction is as follows:

p	q	p ^ q
T	T	F
T	F	T
F	T	T
F	F	F

ii. Inclusive (Weak) Disjunction:

- The inclusive sense of "or" is "Either A or B (or both)."
- In symbolic logic, we use the wedge (v) in the weak sense to represent the disjunction regardless of which sense of 'or' is intended. sense of "or" is "Either A or B (but not both)'

The inclusive sense of "or" is "Either A or B (but can be both)." For example,

a. "John is at the library, or John is studying."

b. "Students are either intelligent or hardworking".

In the first example, both disjuncts can be true because John can be either in the library or at the same time he is studying there. In the second example, both disjuncts can be true because students can be both hardworking and intelligent. The first proposition may be symbolically represented as 'L v S', and the second proposition may be symbolically represented as 'I v H'. The Latin word 'Vel' expresses the inclusive sense of the word 'or'; usually, the first letter of the 'Vel' (v) symbolises the 'or' in the inclusive sense. If 'p' and 'q' are two statements, their weak or inclusive disjunction is written as (p v q). Inclusive disjunction is also a truth-functional compound statement, and the symbol 'v' (vel) is a truth-functional connective.

Truth Table for Weak Disjunction:

- Four possible sets of truth values in weak disjunction

Given any two statements, p and q, they can have just four possible sets of truth values. The four possible cases of inclusive/ weak disjunction can be exhibited as follows:

In case p is true and q is true, p v q is true;

In case p is true and q is false, p v q is true;

In case p is false and q is true, p v q is true;

In case p is false and q is false, p v q is false.

A weak or inclusive disjunction is false only if both of its disjuncts are false. In all other cases, disjunction is true. The following truth table represents the weak disjunction.

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Partial Common Meaning of Disjunction:

The disjunction that claims that ***at least one of its disjunct must be true*** is called an ***inclusive*** or ***weak*** disjunction.

The disjunction that claims that ***at least one of its disjunct must be true*** and *at* **least one of its disjunct must be false** is called an ***exclusive*** or ***strong*** disjunction.

- The shared meaning between the inclusive types of disjunction is that at least one of the disjunct is true

The claim common to both inclusive and exclusive types of disjunctions is that '***at least one of its disjunct must be true'.*** This is known as the *partial common meaning* of both types of disjunctions. The ***partial common meaning*** that **at least one disjunct is true** is the whole meaning of an inclusive disjunction and a part of the meaning of an exclusive disjunction.

At times, it is difficult or even impossible to differentiate the sense in which a disjunction is used. Symbolic logic interprets both types of disjunctions (unless specified) with a partial common meaning regardless of the exact sense in which they are used in statements. This is so because this partial common meaning is sufficient to apply all rules related to disjunction towards validating an argument. So, all disjunctions with **at least one of their disjuncts true** are treated as true disjunctive statements.

c) Negation:

- Negation is formed by inserting a 'not' into the original statement

Negation is a truth-functional operator. The negation, contradiction, or denial of a statement is formed by inserting a 'not' into the original statement. One can express the negation of a statement by prefixing the phrase 'it is false that' or 'it is not the case that'. e.g. 'It is not the case that today is Monday' is the negation of the statement 'Today is Monday.'

The symbol '~' (curl) or tilde is used to form the negation of a statement.

Thus, if we symbolise the statement 'Today is Monday' using the letter 'M', then its negation may be symbolised as ~M. ~M here is read as 'Today is not Monday'/ 'It is not the case that today is Monday'/ 'It is false that today is Monday' etc. If the statement 'Today is Monday' (p) is true, then its negation 'Today is not Monday' (~p) is false; and if the statement 'Today is Monday' (p) is false, then its negation 'Today is not Monday' (~p) is true. In other words, the negation of a true statement is false, and the negation of a false statement is true. It may be expressed in the following truth table.

p	~p
T	F
F	T

d) Implication:

- A conditional statement is a compound statement that combines two propositions using the expression 'if…..then….'

When two statements are combined by using the phrase 'if...then…', the resulting compound statement is a conditional statement. It is also called a hypothetical or an implicative statement. The compound statement 'If you study well, then you will get high marks' is a conditional statement. In a conditional statement, the part between 'if' and 'then' is called *antecedent*, and the part that follows 'then' is called *consequent*. In the given example, 'If you study well' is the antecedent, and 'You will get high marks' is the consequent. There are various ways in which implication is used. Some such instances are shown below.

- Different ways of using implication

- If all metals are conductors and copper is metal, then copper is the conductor. Here, the antecedent implies the consequent ***logically.***
- If a geometrical figure is three-sided, then it is a triangle. Here, antecedent and consequent have a ***definitional relationship***.
- If you drop a glass on the floor, then it breaks. Here, antecedent and consequent are ***causally related***.
- If Trump is a peace-loving dictator, then the sun will rise in the West.

Here, there is ***no connection*** between the antecedent and the consequent. However, we know that the consequences can never be true. A conditional claims that it is impossible for its antecedent to be true and its consequent to be false. Here, since the consequent is known to be false, the conditional is used to deny the antecedent in a tricky way.

- A conditional claim that its antecedent cannot be true and that it is consequently false

In all four cases above, the implicative statements claim that 'if its antecedent is true, then its consequent must also be true'. Irrespective of the various ways the implication is used, this is the **partial common meaning** of all implications. An implication is true if it satisfies this condition, or else it is false. That is, **if the antecedent of an implication is true and its consequent is false, then the statement is false**. Alternatively, if an implicative statement has a true antecedent and a false consequent, then the statement is false. Implication in this sense is called Material implication.

The **symbol used for implication is ' ⊃' (horseshoe)**. The symbolic representation of 'if p then q' is (p ⊃ q). It should be noted that the material implication symbol is a truth-functional connective, like the conjunction and disjunction.

Truth Table for Implication

We have seen that a conditional statement is false only if its antecedent is true and its consequent is false; in all other cases, the conditional statement is true. Any conditional statement if p then q is known to be true in case the ~ (p. ~ q) is known to be true. So, we can take (p ⊃ q) as an abbreviation of ~ (p . ~ q). For example, the statement 'If you drop glass on the floor, then it breaks' may be symbolised as D ⊃ B. D ⊃ B is false if you actually drop a glass on the floor and it does not break, or it is the case that D . ~B

D ⊃ B is true if it is not false or when it is not the case that (D . ~B).

D ⊃ B is true if ~ (D . ~B) is true. Or we may say that DG ⊃ BD is equivalent to

~(D . ~B). Or p ⊃ q is equivalent to ~ (p . ~ q).

The various truth values for 'p', 'q', and 'p ⊃ q' may be tabulated as follows:

p	q	~q	p . ~q	~(p . ~q)	p ⊃ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Leaving the derivation aside, looking into the four possible combinations, we may simplify the above truth table as follows.

The four possible cases can be exhibited as follows:

- Four possible sets of truth values in implication

In case p is true and q is true, p ⊃ q is true;

In case p is true and q is false, p ⊃ q is false

In case p is false and q is true, p ⊃ q is true;

In case p is false and q is false, p ⊃ q is true.

The truth table for material implication is as follows:

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

Conditional statements can be expressed in various ways.

A statement of the form 'if p, then q' could equally well be expressed as:

a. if p, q
b. q if p
c. that p implies that q
d. that p entails that q
e. p only if q
f. that p is a sufficient condition that q
g. that q is a necessary condition that p etc.

- Different ways of expressing the conditional statements

Any of these formulations will be symbolised as p ⊃ q.

e) Bi-conditional (Material Equivalence):

Two statements are said to be materially equivalent or equivalent in truth value when they have the same truth value. We symbolise material equivalence by inserting the symbol '≡' (three bar/ triple bar) between the two statements. Material equivalence is also known as bi-conditional.

- Two statements are materially equivalent when they have same truth value

For example, if p and q are two statements, their material equivalence p ≡ q is true when both p and q are true or when both p and q are false.

The statement p ≡ q is read as 'p if and only if q'. As the expression shows, it is a combination of 'p if q' and 'p only if q'; that means it is a combination of 'q ⊃ p' and 'p ⊃ q'.

That is to say that p ≡ q is the same as (p ⊃ q). (q ⊃ p) or

i.e. (p ≡ q) ≡ [(p ⊃ q) . (q ⊃ p)]

It is because the expression 'p ≡ q' involves two conditionals 'p ⊃ q' and 'q ⊃ p' that it is called biconditional.

Truth Table for Bi-conditional:

Given any two statements, p and q, they can have just four possible sets of truth values. The four possible cases can be exhibited as follows:

- When both the component statements of a material equivalence have the same truth value, the statement is true, or else it is false

In case p is true and q is true, p ≡ q is true;

In case p is true and q is false, p ≡ q is false

In case p is false and q is true, p ≡ q is false;

In case p is false and q is false, p ≡ q is true.

The truth table for Bi-conditional is as follows:

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

Truth Table for Material Equivalence

The logical operator material equivalence is variously expressed as:

a. “If and only if”

- Example: You will get food **if and only if** you give money.

b. “Sufficient and necessary condition”

- Example: Giving money is a **sufficient and necessary condition** to get food.

All these are invariably symbolised as p ≡ q.

2.1.4 Symbolization of Compound Statements: Some points to be remembered

1. It should be noted that the words ‘but, yet, also, still, although, however, nevertheless, and so on, and comma, semicolon, etc., can also be used to conjoin two statements, and they can all be represented by the ‘dot’ symbol.

2. While using ‘both…not’ and ‘not…both’, keep the following things in mind:

‘Arun and Betty will not both be elected’ would be symbolised as ~ (A . B)

‘Arun and Betty will both not be elected’ will be symbolised as ~A. ~B

3. The negation of a disjunction is often expressed by using the phrase ‘neither nor’.

‘Neither Arun nor Betty will be elected’ can be symbolised in two different ways:

~ (A v B)

~A . ~B

because the statement also means ‘Arun won’t be elected and Betty won’t be elected.

4. The word 'unless' can also be used to form a disjunction of two statements. 'Unless there is a strike, the classes will be held' is identical to the statement 'Either the classes will be held or there will be a strike'. This can be symbolised as 'C v S' and, more generally, as 'p v q'.

5. Conditional statements can be expressed in various ways:

   a. 'if p, q',

   b. 'q if p',

   c. 'that p implies that q',

   d. 'that p entails that q'

   e. 'p only if q'

   f. 'that p is a sufficient condition that q',

   g. 'that q is a necessary condition that p',

   and any of these formulations will be symbolised as p ⊃ q.

2.1.5 Summary of Truth Functional Connectives

	Truth function	Connective Symbol	Phrases
a	Conjunction	. (dot)	And, but, however, moreover, furthermore, yet, still, also, nevertheless, although etc.
b	Disjunction	v (wedge or *vee*)	Either...or, or, unless
c	Implication	⊃ (horseshoe)	If...then, implies, only if, is a sufficient condition for, **(q ⊃ P if p is a necessary condition for P)**
d	Equivalence	≡ (three bar)	If and only if, materially equivalent, logically equivalent
e	Negation	~ (curl, tilde)	Not, it is not the case that

Summarized Overview

The present Unit furnishes a comprehensive outline of sentences, propositions, and statements, underscoring their grammatical, logical, and truth-functional elements. It defines a sentence as a meaningful arrangement of words that must adhere to grammatical rules to convey a precise meaning or proposition. Propositions, central to the discussion, are presented as the meaningful content of sentences, capable of being true or false. It is highlighted that only declarative or factual statements are propositions. It distinguishes between simple and compound statements, with the latter containing more than one simple statement and being classified further based on their logical connections (conjunction, disjunction, implication, and biconditional). The concept of a truth-functional compound statement is introduced, emphasising that the truth value of a compound statement depends on the truth values of its component statements. However, it notes that not all statements with multiple parts are truth-functional compound statements.

Self-Assessment

1. What are the differences between a sentence and a proposition?
2. What is meant by a simple proposition?
3. Write a note on compound propositions.
4. What are the four types of compound propositions?
5. What is meant by truth-functional compound statements?
6. What is meant by truth-functional connectives?
7. Draw the truth table of conjunction.
8. What is meant by conjunction?
9. What is the implication of the dot (.) symbol?
10. What are two types of disjunctions?
11. What is meant by implication?
12. Write the meaning of the horseshoe symbol.
13. What is the meaning of the symbol '≡'.
14. Write a short note on the symbol of negation.

Assignments

1. Symbolise the following propositions using suitable abbreviations.

   a. New Delhi is the capital of India and Droupadi Murmu is the president of India.

   b. If it rains, then we shall go for a picnic.

   c. You will catch the train if and only if you reach the station.

   d. Neither Ram nor Mohan was elected in the assembly election.

   e. A student is suspended from the college if his behaviour is indecent.

   f. X will go on tour if and only if K also goes on tour.

2. Symbolise the following propositions using suitable variables.

   a. If I do not go to Delhi, then I will attend my friend's marriage and give her a gift.

   b. If mathematics is difficult, then you will pass only if you work hard.

   c. If hydrogen and oxygen are mixed in the required proportion, water is produced.

   d. Unless it rains in time, the harvest will not be good.

   e. It is not true that neither John nor James won.

   f. He is both bad and a coward.

   g. I cannot drive a car if it is dark.

   h. Ramu was annoyed, but he kept quiet.

3. If A and B are known to be true, and X and Y are known to be false, but the truth values of P and Q are not known, of which of the following statements can you determine the truth value?

   a. ~ (A v B)

   b. ~A . ~B

   c. ~A . P

d. P . X

e. P v ~Y

f. [(P . Q). (~A v X)]

g. [P . (Q v X)] v ~[(P . Q) v (P . X)]

h. Q v ~ (P . Q)

i. ~P v (~ A v P)

j. P v ~Y

4. Explain the nature of different types of truth-functional compound statements.
5. Why is negation called a truth modifier?
6. Explain the concept of material implication.

Suggested Reading

1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
2. Copi, I. M., & Cohen, C. (1990). *Introduction to Logic* (9th ed.). Macmillan Publishing
3. Jain, Krishna (2013). *Textbook of Logic* (5th ed.). D.K Printworld Pvt. Ltd.
4. Klenk, V. (2007). *Understanding Symbolic Logic* (5th ed.). Prentice Hall. River, New Jersey

Space for Learner Engagement for Objective Questions

Learners are encouraged to develop objective questions based on the content in the paragraph as a sign of their comprehension of the content. The Learners may reflect on the recap bullets and relate their understanding with the narrative in order to frame objective questions from the given text. The University expects that 1 - 2 questions are developed for each paragraph. The space given below can be used for listing the questions.

UNIT 2
Statement Forms: Tautology, Contradictory and Contingent

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- understand how substitutions of statement variables produce valid statements
- attain the ability to construct truth tables for compound statements and determine their nature
- develop critical thinking skills to evaluate complex arguments using the techniques learned
- enhance logical reasoning skills through practice in creating and interpreting truth tables

Background

Statements or propositions are the building blocks of arguments. In the previous block, we have seen that the truth of the statement, along with the validity of arguments, determine the truth of the conclusion being claimed. This poses questions such as: how can we determine whether an argument is valid or not? How are the truth/ falsehood of statements determined? The techniques used to determine the validity/ invalidity of arguments are known as decision procedures. There are various kinds of decision procedures, such as the 'truth table method', 'formal proof of validity', 'conditional proof', indirect proof' etc. The truth table method for validation of arguments will be taken up in the following unit, and the other methods will be used in the next block of the text. The answer to the question, 'Are the truth/ falsehood of all statements based on facts?' is 'Not always'. As far as simple statements are concerned, they are often statements of facts, and their truth is determined by the outside world. To put it in other words, simple statements often are empirical statements. There, however, are some compound statements whose truth is determined from within the statement itself. They are formal statements. The truth table method, which can be used to validate arguments, can also be used to determine the nature (whether it is always true/ false or something else) of formal statements. So, let us first explore the truth table method.

Keywords

Truth table method, Statement form, Specific form of a statement, Tautology, Contradictory, Contingent

Discussion

2.2.1 Truth Table Method

- The truth table method is the most basic method to examine both the nature of statement forms and test the validity of arguments

The truth table method is the most basic and most popular method to examine both the nature of statement forms and test the validity of arguments. We have seen in the previous unit that the truth value of a truth-functional compound statement is determined by the truth value of its component statements. The nature of a compound statement can be checked by examining all the possible truth value combinations of its component statements. In the same way, the validity of an argument can be checked by examining all the possible truth values of its premises and its conclusion and seeing that nowhere does the conclusion go false when its premises are true. In both cases, examining the nature of compound statements and the validity of arguments, in order to list all the possible combinations of truth values of statements and to carry out further examination, we need to construct the truth table. Let us see how to do this.

- The total number of rows is equal to the number of possible combinations of truth values for the simple proposition

In constructing a truth table, the first factor that must be determined is the number of rows in the table. The total number of rows is equal to the number of possible combinations of truth values for the simple propositions. Where *R* designates the number of rows and *n* the number of *different simple* propositions (the total number of variables), the number of rows may be computed by the following formula: $R = 2^n$.

In the truth table technique, all possible combinations of truth values of statement variables are listed in the first columns. If there is only one variable, then it can have two values; if there are two variables, then they together can have four combinations of truth values. For three variables, eight, for four, sixteen and so on.

Some Points to Remember:

Before starting to construct the truth table, the following things need to be noted:

- The number of rows in a truth table is to be calculated based on the number of statement variables.
- The total number of columns will be the sum of the number of statement variables

a. The number of rows is to be calculated based on the number of statement variables.

b. Determine the number of columns. The total number of columns will be the sum of the number of statement variables and the number of truth-functional connectives (without repetition) involved.

c. The truth values are assigned as follows: The last variable is assigned alternate Ts and Fs from the top to the bottom of the column. The variable prior to last is assigned pairs of Ts and Fs from top to bottom of the column. The third variable from the right is assigned quadruples of Ts and Fs.

No. of Simple Statements	Variables to Represent Simple Statements	Calculation of Number of Rows		Total Number of Rows
1	p	2^1	2	2
2	p, q	2^2	2 x 2	4
3	p, q, r	2^3	2 x 2 x 2	8
4	p, q, r, s	2^4	2 x 2 x 2 x 2	16
5	p, q, r, s, t	2^5	2 x 2 x 2 x 2 x 2	32

Table showing the relation between the number of simple statements and the number of rows in a truth table.

Some examples of the relation between the number of variables and distribution of truth values.

1 Variable

p
T
F

2 Variables

p	q
T	T
T	F
F	T
F	F

3 Variables

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Example for constructing a truth table for a statement to analyse its nature:

Consider the statement (p v q) ⊃ r

- Firstly, we need to determine the number of rows and assign truth values for the variables.
    - This form has 3 variables; so, the number of rows will be 2^3 (2 x 2 x 2) = 8.
    - The last variable, 'r', will have alternate Ts and Fs filed from the top.

    - The variable second from last, ‘q’, will have pairs of Ts and Fs assigned from the top of the table.
    - The first variable, ‘p’, will have quadruplets of Ts and fs assigned from the top of the table.
- Secondly, we need to determine the number of columns and name each column.
    - The total number of statement variables is 3.
    - The total number of truth-functional connectives without repeating statements is 2.
    - Therefore, the number of columns required is 3 + 2 = 5

Now, we can construct the truth table as follows:

- In order to construct a truth table for the statement (p v q) ⊃ r, other than assigning truth values to the simple statements p, q and r, we need to calculate the truth values for ‘p v q’ and ‘(p v q) ⊃ r’ also.

p	q	r	p v q	(p v q) ⊃ r
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

The assignment of truth values for compound statements in the above tables will be taken up later.

2.2.2 Statement Forms:

Definition:

A *statement form* may be defined as any sequence of symbols containing statement variables, such that when statements are substituted for the statement variables- the same statement being substituted for every occurrence of the same statement variable throughout - the result is a statement.

It is conventional to use the letter 'p' as the first statement variable; the letter 'q' shall be the second, 'r' the third, and so on.

Thus,

- p v q is a statement form called "disjunctive statement form".
- p . q is "conjunctive statement form".
- p ⊃ q is "conditional statement form".
- ~ p is called a "negation form" or "denial form".

When we replace a statement for a statement variable of a statement form, then the statement is treated as a substitution instance of that statement form. So, any statement that results from substituting statements for the statement variables of a statement form is said to have that form or to be a substitution instance of it.

- Any statement that results from substituting statements for the statement variables of a statement form is said to have that form or to be a substitution instance of it

Thus,

- "I will either go for higher studies or employment" is a substitution instance of the form p v q.
- "If I go for employment, I can look after my family" is a substitution instance of the form p ⊃ q.

Consider the statement:

"If you want to be happy in life, either you study hard and do research or get a good job."

This may be symbolised as: H ⊃ [(S . R) v J]

The form of the statement is: p ⊃ [(q . r) v s]

Specific Form of a Statement:

The specific form of a statement is that statement form obtained by substituting each simple statement with statement variables.

For example, where A, B, and C are different simple statements, the compound statement A ⊃ (B v C) is a substitution instance of statement form such as

a. p ⊃ q (by considering 'B v C' as one statement.)

b. p ⊃ (q v r).

- The specific form of a statement is that statement form obtained by substituting each simple statement with statement variables

However, only p ⊃ (q v r) is the specific form of the statement.

Take another example: statements like '(F . G) v (M . H)' and '(K . L) v (R . S)' are substitution instances of :

a. p v q

b. p v (q . r).

c. (p . q) v (r . s)

However, only c) (p . q) v (r . s) is the specific form of the statement.

2.2.3 Types of Statement Forms

- Three types of statement forms: tautologies, contradictories, and contingent

All statements are either true or false. The truth or falsity of statements is determined by the sense or content they are meant to convey. However, at times, the statement form may determine whether the statement is true or false. Some statements can never become false or are always true (tautologies), and some others can never become true, or they are always false (contradictories), and the rest, which are neither tautology nor contradictory, are known as contingent. So, there are three types of statement forms: tautologies, contradictories, and contingent.

a) Tautology:

A statement form is called tautology if it cannot become false. In other words, a statement with only true substitution instances is considered tautologous or tautology. Any substitution instance of tautology must be tautology. For example, "Marconi invented the radio, or he did not". By using an abbreviation, this statement can be symbolised as M v ~M. The specific form of 'M v ~M' is 'p v ~p', which may be proved as a tautology by the following truth table.

M	~M	M v ~M
T	F	T
F	T	T

- A statement form with only true substitution instances is called tautologous or tautology

The third Column (the main column) contains only true substitution instances. Hence, it is a tautology.

Take another statement form, (p ⊃ q) v p can be checked whether tautology or not using the truth table below.

p	q	p ⊃ q	(p ⊃ q) v p
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

We can see that (p ⊃ q) v p is a tautology as the fourth column (the main column) has only true substitution instances. Therefore, this statement is also a tautology.

b) Contradictory:

A statement form is called contradictory if it cannot be true in any of its substitution instances. In other words, a statement that has only false substitution instances is said to be a contradiction. Every substitution instance of this contradictory statement form must be false and thereby are contradictory statements.

For example, consider the statement:

'Tom is an honest man, and he is not.'

This may be symbolised using the abbreviation H. ~H.

This is a contradictory statement because Tom cannot be honest and dishonest at the same time.

'p. ~p' is the specific form of '(H. ~H) ', and it may be proved as contradictory by the truth table method as follows:

p	~p	p . ~p
T	F	F
F	T	F

- A statement form that has only false substitution instances is said to be a contradiction

The third column (the main column) has only false substitution instances. Therefore, this statement is a contradictory one. In other words, the truth table for this statement shows that the statement cannot be true for any of its substitution instances. If the statement is contradictory, then any statement of the form (p . ~p) is also contradictory.

Let us use the truth table method to prove the statement form (p ⊃ q) . (p . ~q) as contradictory.

p	q	~q	p ⊃ q	p . ~q	(p ⊃ q) . (p . ~q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

Since the last column (main column) of the table has only false substitution instances, the statement (p ⊃ q) . (p . ~q) is a contradictory one.

c) Contingent:

- Contingent statements have at least one true and one false substitution instance

Statements and statement forms that are neither tautology nor contradictions are said to be contingent. For instance, p, ~p, p v q, p. q are contingent statement forms. Contingent statement forms have at least one true and one false substitution instance. For example, 'p ≡ q' is a contingent statement because it has both T's and F's in the main column of its truth table.

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

Logical Equivalence as Tautological Statements:

We have already discussed in the previous chapter that two statements are said to be materially equivalent when they have the same truth value. We symbolise the statement that they are materially equivalent by inserting the symbol '≡' between them. Material equivalence is also known as bi-conditional. Here, in the case of material equivalence, the two statements merely happen to have the same truth value.

Let us discuss the notions of logical equivalence (tautologous biconditional) and De Morgan's theorem as an example of logical equivalence.

'Sun rises in the east' is always true; Is it a tautology?

Suppose that you are in an air-conditioned smart classroom. Your friend says that:

It is raining outside. (a)

Now, to determine whether she is right or not, you need to slide the window curtain and look outside to see if it is raining.

After some time, another friend of you says:

It is not raining outside. (b)

As you did earlier, you need to look outside now to confirm whether your friend is right or not.

Again, it so happened that your third friend, after some time, says:

It is either raining outside or it is not raining outside. (c)

Your friend Ancy, being a smart student of Symbolic logic, instead of looking outside to determine whether that friend is right or not, did the following:

Ancy:

Let us represent *It is raining outside* using the letter 'R'.

Now, *It is not raining outside* is to be represented as '~R'.

If so, *It is either raining outside or it is not raining outside* must be represented as 'R v ~R'.

Without looking outside, let us construct a truth table to decide the nature of 'R v ~R'.

R	~R	R v ~R
T	F	T
F	T	T

Now, we know that *It is either raining outside or it is not raining outside* (R v ~R) is true despite what is happening outside.

Statement (a) is an empirical statement. It speaks about a fact in the world, which may be true or not.

Statement (b) too is an empirical one in the same way as (a).

Whereas statement (c) is a **tautology**, it **speaks about itself, the relation between its elements constitutes its truth**. It is not speaking anything about the world.

See how two statements speaking about the world, when joined together form a statement that is not about the world!

The same is the case with contradictories. **Tautology and contradictories are formal statements; their truths are determined within themselves by the relationship between their elements**.

Statements like the one given in the title, 'Sun rises in the east', even though always true are not tautologies. This is because they are empirical statements.

Wittgenstein says:

Propositions show what they say: tautologies and contradictions show that they say nothing.

A tautology has no truth-conditions, since it is unconditionally true: and a contradiction is true on no condition.

Tautologies and contradictions lack sense. (Like a point from which two arrows go out in opposite directions to one another.) (For example, I know nothing about the weather when I know that it is either raining or not raining.) 4.461

Tautologies and contradictions are not, however, nonsensical. They are part of the symbolism, much as '0' is part of the symbolism of arithmetic. 4.462

Tautologies and contradictions are not pictures of reality. They do not represent any possible situations. For the former admit all possible situations, and the latter none.

In a tautology the conditions of agreement with the world- the representational relations- cancel one another, so that it does not stand in any representational relation to reality. 4.463

Tractatus Logico Philosophicus

- Logical equivalent statements can be used interchangeably without affecting the truth value

There is another class of statement forms known as logical equivalent statement forms that are of much use in symbolic logic. Two statement forms are logically equivalent when they have the same sense. They can be used interchangeably without affecting the truth value of the whole statement containing them.

While the materially equivalent statements are equivalent by virtue of their empirical status, the logically equivalent statements are equivalent by their formal structure.

Two statements are logically equivalent when the biconditional that expresses their material equivalence is a tautology. That is to say that if two statements are logically equivalent, then they have the same truth value for every substitution instance. So, all logically equivalent statements are biconditionals, but all biconditionals are not logically equivalent.

- Only tautologous bi-conditionals are called logical equivalence

Two statements are logically equivalent only when it is absolutely impossible for the two statements to have different truth values. Therefore, logically equivalent statements have the same meaning and may be substituted for one another in any truth-functional context without changing the truth value of that context. So, we can say that only tautologous bi-conditionals are called logical equivalence. Thus, the principle of double negation, expressed as the bio-conditional p ≡ ~ ~P, is proved to be tautologous. The following truth table proves the logical equivalence of p and ~ ~p.

p	~p	~ ~p	p ≡ ~ ~P
T	F	T	T
F	T	F	T

Consider two statement forms, 'p ⊃ q' and '~p v q'. These statement forms can be shown to be logically equivalent by showing that their material equivalence (p ⊃ q) ≡ (~p v q) is a tautology. This is done as below:

p	q	p ⊃ q	~p	~p v q	(p ⊃ q) ≡ (~p v q)
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

De-Morgan's Theorems:

De-Morgan's theorems are another example of logical equivalence (tautologous biconditional). They are known as De Morgan's Theorems, after the famous English logician Augustus De Morgan. There are two components in this theorem, and they are as follows;

a. The negation of the conjunction of two statements is logically equivalent to the disjunction of their negations.

For example, suppose that your friend tells you:

i. “I did not have both tea and coffee’”.

What does this mean?

It means that,

ii. she either did not have tea or she did not have coffee.

Both i) and ii) have the same meaning.

i) can be symbolised as ~(T . C)

ii) can be symbolised as ~T v ~C

Now, if both i) and ii) have the same meaning,

~(T . C) ≡ (~T v ~C) must be true. Then, its form

~ (p . q) ≡ (~p v ~q) must also be true.

b. The negation of the disjunction of two statements is logically equivalent to the conjunction of their negations.

For example, suppose that your friend tells you:

i. “I did not have either tea or coffee’”.

What does this mean?

It means that,

ii. she did not have any of the two: tea or coffee. This means that she did not have tea, and she did not have coffee.

Both i) and ii) have the same meaning.

i) can be symbolised as ~(T v C)

ii) can be symbolised as ~T. ~C

Now, if both i) and ii) have the same meaning,

~(T v C) ≡ (~T . ~C) must be true. Then, its form

~ (p v q) ≡ (~p . ~q) must also be true.

These two expressions,

a. The negation of the conjunction of two statements is logically equivalent to the disjunction of their negations.

~ (p . q) ≡ (~p v ~q)   and

b. The negation of the disjunction of two statements is logically equivalent to the conjunction of their negations.

~ (p v q) ≡ (~p . ~q)   are known as De-Morgan's theorems.

Thus, to prove the first part, ~ (p . q) ≡ (~p v ~q), we have to show that the material equivalence of ~ (p . q) and (~p v ~q) is a tautology. This is shown by the truth table given below.

p	q	p . q	~ (p . q)	~p	~q	~p v ~q	~ (p . q) ≡ (~p v ~q)
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

To prove the second part, we have to show that the material equivalence of ~ (p v q) and (~p . ~q) is a tautology. This is shown by the truth table given below.

p	q	~p	~q	p v q	~(p v q)	~p . ~q	~ (p v q) ≡ (~p . ~q)
T	T	F	F	T	F	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	T

Summarized Overview

In this unit, we have stated that a truth table is a method for deciding the nature of formal statements and evaluating arguments. We have also seen the basics of truth table construction. The compound statements can be classified either as a tautology, contradictory or contingent by examining all the possible truth value combinations of its component statements, and the correctness or validity of an argument can be checked by reviewing all the possible truth value combinations of its component statements (premises and its conclusion). We have illustrated the nature of different types of statement forms, such as tautology, contradiction and contingent. In connection with this, the present unit also analysed the concept of tautologous biconditional or logical equivalence and showed how it is different from material equivalence. We also introduced De-Morgan's theorem as an example of tautologous biconditional or logical equivalence.

Self-Assessment

1. What is the significance of the truth table method?
2. What is the significance of logical equivalence?
3. Identify the formula for computing the number of rows in a truth table.
4. Write a short note on how to calculate the total number of rows and columns in a truth table.
5. What is a statement form?
6. Write a note on the substitution instance of the statement.
7. What is meant by the specific form of a statement?
8. What is tautology?
9. Are all biconditionals tautologies? Explain.
10. Is De-Morgan's theorem a tautology? How?
11. What is the difference between material equivalence and logical equivalence?
12. What is meant by contradictory?
13. What is meant by contingent?

Assignments

1. Examine the nature of the truth table method as a decision procedure.

2. Differentiate between statement forms and specific forms of a statement with examples.
3. Explain the differences between the three kinds of statement forms with examples.
4. Pick out the statement forms and specific forms of statements from the given choices.

i. Y v (K ≡ H)

a. p v q

b. p v (q ≡ q)

c. p v (q ≡ r)

d. p v p

e. p v ~p

ii. S ⊃ (U . H)

a. p ⊃ q

b. p ⊃ (q . q)

c. p ⊃ (q . r)

d. p ⊃ p

e. p ⊃ (~p . q)

iii. Y . (K ⊃ H)

a. p . q

b. p. (q ⊃ q)

c. p. (q ⊃ r)

d. p . p

e. p . (~p ⊃ q)

5. Use the truth table method to find out which of the following propositional forms are tautologous, contingent or self-contradictory.

a. (p ⊃ q) . ~(p . q)

b. (p v q) v ~(p . q)

c. (p ⊃ q) . ~(p ⊃ q)

d. (p ≡ q) ⊃ (p v q)

e. [(p ⊃ q) . p] v [(p v q) ≡ (p . ~q)]

f. [(p v q) ⊃ r] ⊃ (p ⊃ r)

g. [~p . (q v r)] ⊃ ~(q v r)

Suggested Reading

1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
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3. Jain, Krishna (2013). *Textbook of Logic* (5th ed.). D.K Printworld Pvt. Ltd.
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UNIT 3
Truth Table Method for Evaluating Arguments

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- identify and define the premises and conclusion of an argument.
- differentiate between argument forms and specific forms of arguments.
- symbolise arguments using statement variables.
- construct and analyse truth tables to validate argument forms.

Background

An argument is a group of propositions or statements in which one or more of them support or provide evidence for the truth of another statement. The premises of an argument are the propositions which provide the grounds or reasons for accepting the conclusion. The derived proposition is known as the conclusion. Propositions, which are the ground of reasoning, are called premises. For example,

All flowers are beautiful.

Rose is a flower.

Therefore, rose is beautiful.

In this example, the first two statements are called premises, and the last statement (Therefore, rose is beautiful.) is called the conclusion of the argument.

An argument is a set of statements, one of which (the conclusion) is claimed to be supported by the others (the premises). While statement forms are evaluated as tautologies, contingent, and self-contradictory, argument forms are evaluated as valid or invalid.

The validity/ invalidity of an argument is the formal nature of that argument. This does not follow from the truth or meaning of their premises or conclusion, but the way premises and conclusion in that argument are related. This relation between the premise and conclusion of an argument is determined by the structure of that argument. Hence, through validating the structure, we can validate an argument.

The validation can be done by examining all the possible truth values of its premises and its conclusion and seeing that nowhere does the conclusion go false when its premises are true. To examine the nature of compound statements and the validity of arguments, we need to list all the possible combinations of truth values of statements and carry out further examination. For this purpose, we construct the truth table. In this unit, we use the method of truth table used in the previous unit to assess arguments. Using the truth table method, we can quickly evaluate the lengthy arguments that contain ambiguous words.

Keywords

Premises, Conclusion, Validity, Argument form, Specific form of an Argument, Truth Table Method

Discussion

2.3.1 Argument Form and Specific Form of an Argument

The validity or invalidity of an argument is formal in its nature. To understand the structure of an argument, we need to avoid looking into the propositional contents of the argument. That is to say that we need to get into the argument form.

Argument Form:

- The argument form shows the logical structure of an argument

The *argument form* is an array of symbols that contains statement variables, such that when statements are substituted for the statement variables-the same statement being substituted for every occurrence of the same statement variable throughout-the result is an argument. Thus, the argument;

If I am an actor, then I am famous.

I am not an actor.

Therefore, I am not famous.

which may be symbolised as

A ⊃ F		p ⊃ q
~A	has the form	~p
∴ ~F		∴ ~q

The argument form may be identified by replacing the statements within the argument with statement variables while retaining the logical constants as they are. In doing so, the first statement has to be replaced by the letter 'p', the second by the letter 'q' and so on, as mentioned earlier. The same is done above to get the argument form.

2.3.1.1 Specific Form of an Argument

The *specific form* of a given argument is that argument form from which the argument results by substituting a different *simple* statement for each distinct statement variable. An argument form may be formed by substituting a statement variable for a compound statement. However, a form made in such a way is not the *specific* form of that argument. To get a specific form, each simple statement is to be substituted by a statement variable.

- To get a specific form, each simple statement is to be substituted by a statement variable. By this, we get into the complete structural details of an argument

For example, consider the argument:

Either Raju could go for shopping and get a ready-made shirt, or he could get one tailored.

Raju did not go for shopping.

Therefore, Raju got his shirt tailored.

The above argument and its form are:

Argument	***Form***
S v T	p v q
~S	~p
∴ T	∴ q

The argument with details and its specific form are:

Argument	*Form*
(S. R) v T	(p . q) v r
~S	~p
∴ T	∴ r

2.3.2 Validating Arguments Using Truth Table

We know that the validity/ invalidity of an argument depends on its form. If a form is valid, all arguments of that form are valid. Arguments having invalid forms, on the other hand, are invalid. All arguments belonging to a specific form are said to be the *substitution instances* of that form. In other words, The argument resulted by substituting statements for statement variables in an argument form is called the substitution instance of that argument form.

For example, consider the argument form:

p ⊃ q

p

∴ q

The following are a few substitution instances of this form:

a. If you study hard, you can score high in the examination.

You study hard.

Therefore, you will score high in the examination.

b. If you regularly eat junk food, you will become unhealthy.

You regularly eat junk food.

Therefore, you will become unhealthy.

c. If there are dark clouds, it will rain.

There are dark clouds.

Therefore, it will rain.

- Proving the invalidity of an argument by constructing a counterexample having true premises and false conclusion is called Refutation by Logical Analogy

In order to test the validity of all these arguments, we need only to test the validity of the given form.

In the same way, if a form is invalid, all its substitution instances are invalid. An argument can be shown to be invalid by showing any of its substitution instances having true premises and false conclusions. Such a demonstration shows that the form of the argument itself is invalid.

Consider the earlier example:

a) If I am an actor, then I am famous.

I am not an actor.

Therefore, I am not famous.

We have seen that its form is:

$p \supset q$

$\sim p$

$\therefore \sim q$

Now, we may formulate another example using the same form.

b) If Mother Theresa is an actor, then she is famous.

Mother Theresa is not an actor.

Therefore, Mother Theresa is not famous.

In example (b) above, both the premises are true, and the conclusion is false. Hence, this argument is invalid. Hence, the argument form, too, is invalid. If the form is invalid, the original argument (a) must also be invalid. This method of Refutation by logical analogy or counterexample method shows that the validity of the argument depends on its form.

2.3.2.1 Checking Validity/ Invalidity

To check the validity of an argument, we may check its form for its validity. In doing so, we should check all possible instances where its premises are true and see whether the conclusion becomes false. If there is any such instance, the argument form is invalid, and hence the argument. If there is no instance in which the conclusion is false while the premises are true, then the argument is valid.

- An argument having at least one substitution instance with true premises and false conclusion is an invalid one

We can test the validity of an argument using the truth table method. The truth table method is the most popular method for examining the validity of arguments. In doing so, we should assign all the possible truth values for variables and statements and see whether the conclusion becomes false while the premises are true. We may check the validity of the above argument (a) using its form:

$p \supset q$

$\sim p$

$\therefore \sim q$

p	q	p ⊃ q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

This argument is clearly shown to be invalid, as shown by row number 3, where both the premises are true and the conclusion is false.

Let us take another example:

If you win the lottery, you can either purchase a house or a car. You won the lottery, and you did not purchase the house. Therefore, you purchased a car.

- If an argument has no substitution instance with true premise and false conclusion, then it is a valid argument

This symbolised argument and its form are:

Argument	**Form**
L ⊃ (H v C)	p ⊃ (q v r)
L . ~H	p . ~q
∴ C	∴ r

The truth table for an argument may be constructed either using the symbolised argument (in this example, using the abbreviations L, H and C) or by using argument form. Let us use this example of the symbolised argument itself.

L	H	C	~H	H v C	L ⊃ (H v C)	L . ~H
T	T	T	F	T	T	F
T	T	F	F	T	T	F
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	T	F	F	T	T	F
F	F	T	T	T	T	F
F	F	F	T	F	T	F

Here, both the premises are true only in row number 3, and the conclusion (C) is also true in this row. Since there are no substitution instances for this argument with true premises and false conclusions, the argument is a valid one.

Summarized Overview

The present unit outlines the structure and validity of formal arguments and introduces the concepts of argument forms and specific forms of an argument. An argument form is a structural representation of an argument using statement variables. We have seen that the complete structural details of an argument are represented by its specific form. Since an argument makes the claim of its conclusion based on the truth of its premises, we can show the invalidity of an argument by showing at least one substitution instance in which the premises are true and the conclusion false. This can be done using the truth table method. If there is no substitution instance for an argument having true premises and false conclusion, then the argument is a valid one.

Self-Assessment

1. What is an argument form?
2. What is meant by substitution instance of argument from?
3. What is a specific form of argument?
4. Is an argument with false premises and a true conclusion valid?
5. What is meant by refutation by logical analogy?
6. Must an argument with all true premises be valid?

Assignments

1. Symbolise the following arguments and determine their validity/invalidity by the truth table method.

   a. If cheetahs are fighters or deer, they are weak animals. If deer are not weak animals, then cheetahs are not fighters. Deer are indeed weak animals. Thus, cheetahs are fighters.

   b. If Reshmi is late, she will miss the university exam. If Reshmi misses the university exam, her parents will not be happy. Either Resmi's parents will be happy, or she will not miss the university exam. Therefore, Resmi will not be late.

c. The Republican party cannot win the election unless it drops the prices of essential commodities. If new economic policies are implemented, then the prices of essential commodities will also drop. Either the Republican party does not win the election, or the latest economic policies are implemented.

d. Either the principal does not know the truth, or she is hiding something in the interest of the institution. If she is hiding something in the interest of the institution, then the police should provide necessary security. Therefore, it is not the case that if the principal knows the truth, then the police will not offer her the necessary security.

2. Construct a truth table for the following argument forms and determine their validity/invalidity.

a. $p \supset q$

$\sim p \supset r$

$\therefore q \vee r$

b. $p \vee (q \cdot \sim q)$

$p \supset \sim q$

$\therefore q \cdot \sim p$

c. $(p \vee q) \supset (p \cdot q)$

$p \cdot q$

$\therefore p \vee q$

d. $\sim (p \supset q)$

$q \supset r$

$\therefore \sim r$

3. Illustrate the truth table method for testing the validity of the argument.
4. Differentiate the argument and argument form.
5. Elaborate on the concepts of substitution instance of argument form and the specific form of argument.

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1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
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BLOCK 3
Methods of Deduction

UNIT 1
Rules of Inference and Formal Proof of Validity

Learning Outcomes

Upon completion of this unit, the learner will be able to:

- recognise and understand the common Rules of Inference.
- have a clear understanding of the formal nature of validity and invalidity of arguments.
- apply various Rules of Inference to validate arguments in Symbolic logic.

Background

Logic is the bedrock upon which many aspects of everyday decision-making are built. Reasoning is employed in varying degrees, from simple life situations of choosing a train to reach the University to more complicated academic topics. As seen earlier, the formal aspects of this logical reasoning are handled by Logic. C S Pierce defines logic as the study of the methods and principles that, when used, help to distinguish a correct argument from an incorrect one. This may prompt us to ask the question, how do we differentiate a correct argument from an incorrect one? The answer to this is quite simple, as all you have to keep in mind is that if its conclusion logically follows from its premises, then it can be said to be a valid argument. Imagine a detective trying to solve a mystery using clues to arrive at the truth without making any logical missteps. Here, the clues are the premises, and the fact that is arrived at is the conclusion. The question of validity and invalidity of the process acts as the formal background through which the detective arrives at the truth. To check the formal validity of a group of complex premises, one may use various methods. One of the methods of establishing the validity of an argument is the method of "Formal Proof of Validity". In order to construct the Formal Proof of Validity of arguments, the elementary valid argument forms known as "Rules of Inferences" are employed. This method of Formal Proof of Validity will equip the learner with tools that will help you to reach logically reliable conclusions.

Keywords

Method of Deduction, Rules of Inference, Formal Proof of Validity

Discussion

3.1.1 Formal Proof of Validity

Theoretically, one may use the truth table method, which we have already discussed in the previous block, to validate any argument in propositional logic. Practically, this process is tedious and time-consuming as an argument involves a greater number of component statements. For example, to validate an argument with 5 component statements using a truth table, we need to construct a truth table with 32 rows. A more convenient method of proving validity in this case would be to deduce the conclusions from their premises by a series of shorter, elementary arguments that are already known to be valid. Consider the example.

- Either the CEO has implemented a new policy, or if Smith submitted the report, then Johnson was notified.
- If our internal systems are functioning properly, then if Johnson was notified, Kim received the update.
- If the CEO has implemented a new policy, then our internal systems are not functioning properly.
- Our internal systems are functioning properly.
- Therefore, if Smith submitted the report, then Kim received the update.

- A formal proof of validity demonstrates the validity of a given argument

Such a complex argument demands formal proof of validity as a decision procedure. The formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument or follows from the preceding statement by an elementary valid argument and such that the last statement in the sequent is the conclusion of the argument whose validity is being proved. We can also say that the formal proof is like a detailed recipe that shows how someone got from the ingredients to the final dish using specific steps. Each step must be justified, ensuring that no logical ingredients are missing or misplaced. Here, we must explain what an elementary valid argument means. An elementary valid argument is nothing but a substitution instance of any valid argument form. Ten valid argument forms are employed to establish the formal validity of arguments. These argument forms are collectively called Rules of Inference.

3.1.2 Rules of Inference

The rules of inference are the elementary argument form whose validity can be proved using truth tables. These elementary argument forms can be used to construct a formal proof of validity for a varied range of complex arguments. Like the example that we saw earlier, solving such a complicated argument would be easier with the use of the nine rules of inference.

3.1.2.1 Modus Ponens (M.P.)

- Consequent is derived from the antecedent

Modus Ponens is a hypothetical syllogism. It is the conditional argument where we can infer the consequent of the conditional from the antecedent. This is also known as the method of affirming the antecedent, where the antecedent is affirmed to arrive at the consequent. The Rule of Modus Ponens can only be applied in cases where the major premise is a conditional statement, and the other premise must be the antecedent of the major premise. This rule can be stated as "In every instance, where P implies Q and P is True, Q must be true" This can be summarised as

- Premise 1- if P then Q.
- Premise 2- P is True.
- Conclusion: Therefore, Q is True, which can be symbolised as:

$p \supset q$

p

$\therefore q$

3.1.2.2 Modus Tollens (M.T)

Modus tollendo Tollens or Modus Tollens is in the form of a mixed hypothetical syllogism with two premises and a conclusion. This is an application of a general truth that if a statement is true, so is its contrapositive. This can be demonstrated as:

- Premise 1 - If P then Q
- Premise 2 - It is not Q
- Conclusion - Therefore, it must not be P

- Mixed hypothetical syllogism

The first premise is a conditional "if-then" claim where P implies Q. The second premise is a denial of the consequent of the first premise, that is, the denial of Q. From these two premises. It can be logically concluded that P, the antecedent of the conditional claim, is not the case. This can be stated as "In all the negative instances of the consequent of the conditional, the antecedent should also be negative.". This is symbolised as:

$p \supset q$

$\sim q$

$\therefore \sim p$

3.1.2.3 Hypothetical Syllogism (HS)

It is a pure hypothetical syllogism and is the third rule of inference that we will be looking into. It can be stated that in every instance where p implies q and q implies r, it is the case that p implies r. This is symbolised as:

- Pure hypothetical syllogism

$p \supset q$

$q \supset r$

$\therefore p \supset r$

The consequent of the first premise must be the same as the antecedent of the second premise for the conditional to be valid. Consequently, the antecedent of the first premise must remain as the antecedent of the conclusion and the consequent of the second premise must remain as the consequent of the conclusion.

3.1.2.4 Disjunctive Syllogism (D.S.)

This is a valid elementary form where all the negative instances of p affirm that q is the case; that is, this is the rule that affirms by negating the antecedent of a hypothetical proposition. This can be stated as:

- Either p or q is true
- p is false
- Therefore, q is true. This can be symbolised as:

$$p \vee q$$

$$\sim p$$

$$\therefore q$$

3.1.2.5 Constructive Dilemma (CD)

It is not a foundational rule of inference; rather, it is the combination of modus ponens and disjunctive syllogism. It is the case that if two conditionals of the first premise are true and at least one of their antecedents will be true, then at least one of their consequents must be true as well. So, we can say that if p implies q and r implies s and either p or r is true, then either q or s has to be true.

$$(p \supset q) \cdot (r \supset s)$$

$$p \vee r$$

$$\therefore q \vee s$$

- The combination of modus ponens and disjunctive syllogism

In the argument mentioned above, the first propositions are two conditional propositions: $(p \supset q)$ and $(r \supset s)$. In modus ponens, given the propositions $p \supset q$ and p, we may infer q. Similarly, given $r \supset s$ and r, we may infer s. Therefore, from the conditional propositions $p \supset q$ and $r \supset s$, we can infer either p or r, as well as q or s. These include modus ponens and disjunctive syllogism. In a constructive dilemma, the disjunctives are positive.

On the other hand, in the Destructive Dilemma (DD), the disjuncts are negative, and the inference process is different. It is the inference that if p implies q and r implies s and either q is false or s is false, then either p or r must be false. This can be symbolised as

$$(p \supset q) \cdot (r \supset s)$$

$$\sim q \vee \sim s$$

$$\therefore \sim p \vee \sim r$$

3.1.2.6 Simplification (Simp.)

- Simply from the premises

In the instance where the conjunction of p and q is true, we can derive the conclusion that p is true as both conjuncts must be true for a conjunction to be true.

p • q

∴ p

3.1.2.7 Conjunction (Conj)

- Conjoin two premises

If the case is that the two separate statements are true, then they can be brought together by connecting both statements using the conjunction 'and'.

p

q

∴ p • q

3.1.2.8 Addition (Add.)

- Adding to the premises

Any disjunction must be true if either of its disjuncts is true. That is, given a true statement, we will be able to derive a true disjunction by adding another statement to it. The new disjunctive statement must be true irrespective of the truth value of the new statement, as the first statement is already known to be true. Symbolically:

p

∴ p v q

3.1.2.9 Absorption (Abs.)

- The antecedent is implied by the conjuncts

Every statement implies itself. So, in all true instances of p implying q, it implies both p and q. This rule can be symbolised as:

p ⊃ q

∴ p ⊃ (p • q)

The textbook by I. M. Copi, Fifth Edition, mentions

- A rule of inference helps to determine the validity of an argument in an easy manner

constructive and destructive dilemmas but does not include absorption. However, in later editions, both constructive and destructive dilemmas are combined, and absorption is added as the ninth rule. Absorption becomes necessary in a deductive system where the method of 'Conditional Proof' is not employed. These nine rules are your toolkit for checking the validity of many arguments in Symbolic Logic with minimal difficulty.

3.1.3 Constructing the Formal Proofs of Validity

Now that we have gone through the elementary valid forms, it is very important to understand the application of these rules to construct a formal proof of validity. Let us take the argument:

Either the CEO has implemented a new policy, or if John completed the project, then David received the report. If the internal security system has not broken down completely, then if David received the report, Lilly was notified about the policy change. If the CEO has implemented a new policy, then our internal security system has broken down completely. Our internal security system has not been broken down completely. Therefore, if John completed the project, Lilly would have been notified about the policy change.

Let us assign P for 'The CEO implemented policy', J for 'John completed the project', D for 'David received the report', S for 'internal security system has broken down completely', and L for 'Lilly was notified about the policy change'. Then

- Premise 1- Either the CEO has implemented a new policy, or if John completed the project, then David received the report. This can be symbolised as P v (J⊃ D)
- Premise 2-If the internal security system has not broken down completely, then if David received the report, then Lily was notified about the policy change. This can be symbolised as ~I ⊃ (D ⊃L)
- Premise 3- If the CEO has implemented a new policy, then our internal security system has broken down completely. This can be symbolised as P ⊃ I

- Premise 4 - Our internal security system has not been broken down completely. This can be symbolised as ~I
- Conclusion - Therefore, if John completed the project, then Lily was notified about the policy change. This can be symbolised as ∴ J ⊃ L

Establishing this argument's validity by means of a truth table would require 32 rows, which would require a lot of space and time. We can prove the above argument by inferring the conclusion J⊃L from its premises by a sequence with the help of the ten rules of inference.

1. P v (J⊃D)	
2. ~I ⊃ (D ⊃L)	
3. P ⊃ I	
4. ~I / ∴ J ⊃ L	
5. ~P	3,4 M.T
6. J ⊃ D	1,5 HS
7. D ⊃ L	2, 4 MP
8. J ⊃ L	6,7 HS

Here, we can see that Line 5 is the result of using the rule of Modus Tollens, where lines 3,4 are considered. We infer ~P from lines 3 and 4 by using Modus Tollens, which is to be written on the right side. Similarly, we inferred line 6 by employing the rule of hypothetical syllogism on Line 1 and Line 5. Consequently, we reach the conclusion J ⊃ L, proving the validity of the argument. Exercises for practice will be given at the end of this unit. The student can check their understanding of the rules by attempting the exercise questions.

- The validity of the given argument is determined with the help of rules of inference.

There are two things that one must keep in mind about the rules of inference: One is that they must be followed exactly, and the argument should be in the form of the rule you are choosing. For instance, if you are employing modus Ponens, then it should be in the form of p⊃ q, p / ∴ q. Secondly, the elementary valid form should be applied to the entire line. For example, if we are given [(J•D) ⊃ L] • P, we should consider the whole line and can't directly derive J by simplification.

Summarized Overview

We learned that the validity and invalidity of an argument is an important point of discussion in logic. A valid argument must have its conclusion derived formally from its premises. There are ten valid elementary forms that can be verified by the Truth Table method to check the validity of the argument. We discussed the ten rules in detail. Demonstration of the construction of formal proofs by employing the Rules of Inference is given, according to which the students can attempt the practice exercises given. The rules of inference are powerful tools, as they don't allow any ambiguity or error in arriving at the desired conclusion. Hence, the logical conclusion that is arrived at cannot be doubted. Rules of Inference are redundant and don't allow any kind of replacement; therefore, there's a need for another set of rules that would make the rules of inference more powerful and effective. This set of rules will be discussed in the next unit of this block.

Exercises:

For the following valid arguments, state the Rule of Inference by which its conclusion follows from their premise:

1. A ⊃ B,

   B ⊃ C,

   C ⊃D,

   ~ D,

   A v E/∴ E

2. (B v N) ⊃ (K•L)

   ~K

   ~ M / ∴ ~ B • ~ M

3. (K ⊃ A) • (M ⊃ D)

   ~A /∴ ~K v ~M

4. (M v N) ⊃ (P • Q)

   N /∴ P • Q

5. (A • B) ⊃ (C v D)

   A

   B / ∴C v D

6. (T ⊃ K) • (R ⊃ S)

   S ⊃ D

   D ⊃ T

   R / ∴ T

7. (A v B) • (~D •E)

   A v B ⊃K /∴K • (~D • E)

8. (P ⊃Q) •(R⊃S)

   ~A⊃ ~Q

   A ⊃ B

   ~B /∴ ~P V ~ S

9. A v (B • C)

   A ⊃ P

   ~P / ∴ C

10. A • (B v C)

   A⊃P

   Q / ∴ P•Q

Solutions of the Exercise:

1. 1. A ⊃ B,

    2. B ⊃ C,

    3. C ⊃D,

    4 ~ D,

5 A v E/∴ E

6. A ⊃ C	1, 2 H. S
7. A ⊃ D	6, 3 H. S
8. ~ A	7, 4 M. T
9. ∴ E	5, 8 D S

2. 1. (B v N) ⊃ (K• L)

2. ~ K

3. ~ M /∴ ~B • ~M

4. ~K v ~L	2, Add.
5. ~B • ~N	1, 4, M.T.
6. ~B	5, Simp.
7. ∴ ~B • ~ M	6, 3, Conj.

3. 1. (K ⊃ A) • (M ⊃ D)

2. ~A /∴ ~K v ~M

3 ~A v ~D	2, Add.
4 ∴ ~K v ~M	1, 3 D.D.

4. 1. (M v N) ⊃ (P • Q)

2. N /∴ P • Q

3 M v N	2, Add.
4 ∴ P • Q	1, 3, M.P.

5. 1. (A • B) ⊃ (C v D)

2. A

3. B / ∴ C v D

4. A • B	2, 3, Conj.
5 ∴ C v D	1,4, M.P.

6. 1. (T ⊃ K) • (R ⊃ S)

2. S ⊃ D

3. D ⊃ T

4. R / ∴ T

5. R ⊃ S	1 Simp.
6. S	5,4, M. P.
7. D	2, 6, M. P.
8. ∴T	3, 7, M.P.

7. 1. (A v B) • (~D •E)

2. A v B ⊃K /∴ K• (~D •E)

3. A v B	1, Simp.
4. K	2, 3, M.P.
5.~D •E	1, Simp.
6. ∴ K • (~D • E)	4, 5, Conj.

8 1. (P ⊃Q) • (R⊃S)

2. ~A⊃ ~Q

3. A ⊃ B

4. ~B /∴ ~P V ~ S

5. ~A	3,4, M.T
6. ~Q	2,5, M.P.
7. P ⊃ Q	1, Simp.
8 ~P	7, 6, M.T.
9 ∴ ~ P v ~ S	8 Add.

9. 1 A v (B • C)

2 A ⊃ P

3 ~P / ∴C

4 ~A 2,3, M.T.

5 B • C 1,4, D.S.

6 ∴ C 5, Simp.

10. 1 A • (B v C)

2 A ⊃ P

3 Q / ∴P•Q

4 A 1, Simp.

5 P 2,4, M.P

6 ∴ P • Q 5,3, Conj.

Self-Assessment

1. Define a formal proof of validity.
2. Discuss the rules of inference. And write their names
3. How to construct a formal proof of validity Construct a formal proof to demonstrate the validity of the following argument:

D ɔ E

D . F

∴ E

Assignments

1. F v (G v H)

(G ⊃ I) • (H ⊃ J)

(I v J) ⊃ (F v H)

~F/ ∴ H

2. (A ⊃ B),

C ⊃ D

~B v ~D

~ ~ A

(E • F) ⊃ C / ∴ ~(E•F)

3. (G ⊃ H) ⊃ (I ≡ J)

K v ~ (L ⊃ M)

(G ⊃ H) v ~K

N ⊃ (L ⊃ M)

~ (I ≡ J)/ ∴ ~N

Reference

1. Copi, I. M. (1979). *Symbolic Logic fifth edition*. PHI Learning Private Limited.
2. Lanker, K E, (1967) *An Introduction to Symbolic Logic*, Dover Publications.

Suggested Reading

1. Klenk, V. (2007). *Understanding Symbolic Logic* (5th ed.). Prentice Hall.
2. O'Connor, B. (1993). *An Introduction to Symbolic Logic*. Pearsons.

Space for Learner Engagement for Objective Questions

Learners are encouraged to develop objective questions based on the content in the paragraph as a sign of their comprehension of the content. The Learners may reflect on the recap bullets and relate their understanding with the narrative in order to frame objective questions from the given text. The University expects that 1 - 2 questions are developed for each paragraph. The space given below can be used for listing the questions.

UNIT 2
Rules of Replacement, Conditional Proof and Indirect Proof

Learning Outcomes

After completing this unit, the learner will be able to:

- explain the rules of replacement and their importance in enriching the Rules of Inference.
- gain clarity about the rule of conditional proof and its role in the construction of formal proof of validity.
- identify indirect proof and how it affects proving the validity of an argument.
- apply the rules of replacement, conditional proof and indirect proof in the construction of the formal proof of validity.

Background

The elementary valid argument forms that we have discussed in Unit 1 are powerful instruments in proving the validity of an argument, but they are not sufficient. So, another set of nine rules was added that would help to widen the range, which in turn made the construction of formal proofs easier than just using the rules of inference. This unit deals with methods of deduction that involve the ten rules of replacements, which can be used in conjunction with the rules of inference. The rules of replacement are transformation rules that help in substituting a statement with its logical equivalence. The conditional proof asserts that the antecedent of the statement necessarily leads to its consequent. In other words, proving the antecedent as true allows us to conclude the consequent logically. This helps minimise the steps in the construction of formal proofs. Another method of deduction discussed in this unit is indirect proof, which uses the contradiction of the conclusion to prove its validity.

Keywords

Rules of replacement, conditional proof, indirect proof

Discussion

3.2.1 The Rules of Replacement

- Use the rule of replacement to prove the validity of a part of the propositions in logical equivalence

The inadequacy of the rules of inference in proving the validity of the most obvious elementary truth-functional arguments like A•B/ ∴B mandated a need to use another set of rules. The regulations of replacement are used in cases where any part of a compound statement is replaced by an expression that is logically equivalent to the part that is replaced; the truth value of the replaced statement remains the same as the original statement. For instance, using the principle of double negation, we can assert that P is logically equivalent to ~ ~P. The following are the ten rules of replacement.

- Negation of the conjunction of two statement is logically equivalent to the disjunction of their negations

1. **De Morgan's theorem (De. M.)**: De Morgan's theorem states that (a) The negation of the conjunction of two statements is logically equivalent to the disjunction of their negations. (b) The negation of the disjunction of the two statements is logically equivalent to the conjunction of their negations. These can be represented as:

   ~ (p • q) ≡ (~p v ~q)

   ~ (p v q) ≡ (~p • ~q)

- Commuting variable

2. **Commutation (Com.):** The equivalences of commutation simply mean that the order of component statements of a conjunction, or a disjunction, does not matter. They can be commuted in whichever order happens to appear; the meanings remain the same.

   (p v q) ≡ (q v p)

   (p • q) ≡ (q • p)

3. **Association (Assoc.):** Association states that if three dif-

ferent statements are known to be true, then the disjunction of p with the disjuncts q and r grouped is logically equivalent to the disjunction of p and q grouped to the disjunct r. This remains the case with three disjuncts.

$$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$$

$$[p \bullet (q \bullet r)] \equiv [(p \bullet q) \bullet r]$$

- Conjunction and disjunction are distributed

4. **Distribution (Dist.):** The rule of distribution states that (a) the conjunction of one statement with the disjunction of two other statements is logically equivalent to a disjunction whose first disjunct is the conjunction of the first statement with the second and whose second disjunct is the conjunction of the first statement with the third. (b) the disjunction of one statement with the conjunction of two others is logically equivalent to the conjunction of the disjunction of the first and the second and the disjunction of the first and the third.

$$[p \bullet (q \vee r)] \equiv [(p \bullet q) \vee (p \bullet r)]$$

$$[p \vee (q \bullet r)] \equiv [(p \vee q) \bullet (p \vee r)]$$

- $p \equiv p$

5. **Double Negation (D.N.):** This rule simply means that any statement is logically equivalent to the negation of the negation of that statement.

$$p \equiv \sim\sim p$$

- Transporting the negation of the antecedent and the negation of the consequent

6. **Transposition (Trans.):** Transposition states that any conditional statement is logically equivalent to the conditional statement, asserting that the negation of its consequent implies the negation of its antecedent.

$$(p \supset q) \equiv (\sim q \supset \sim p)$$

7. **Material Implication (Impl.):** The truth table for Implication shows that $p \supset q$ is true whenever either p is false or q is true. This means that $p \supset q$ is logically equivalent to

the disjunction of the two statements 'p is false' and 'q is true'. Symbolically,

$$(p \supset q) \equiv (\sim p \vee q)$$

8. **Material Equivalence (Equiv.):** The rule of Material Equivalence states that two statements are materially equivalent if they both have the same truth value.

- Two statements are materially equivalent

$$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$$

$$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$$

9. **Exportation (Exp.):** When the two statements conjoined imply a third, it is logically equivalent to asserting that if one of those two propositions is true, then the truth of the other must imply the truth of the third.

$$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$$

10. **Tautology (Taut.):** Any statement is logically equivalent to the disjunction of itself with itself, and similarly, any statement is logically equivalent to the conjunction of itself with itself.

- Rules of replacement are built on the value that the expressions involved are logically equivalent

$$p \equiv (p \cdot p)$$

$$p \equiv (p \vee p)$$

3.2.2 Conditional Proof

Many valid arguments are still not covered, even with the replacement rules added to the initial rules of inference. In the cases where there is a conditional statement as the conclusion, both of these rules cannot be employed, and this is where the conditional proof is used. This rule can be explained by the principle of exportation and the correspondence between valid argument form and tautology. Any deductive argument, whether it is valid or invalid, can be expressed in the form of a conditional proposition. The original argument is only valid when the corresponding conditional statement is a tautology. Consider the example,

- The consequent is the conclusion

All A are B

All B is C

∴ All A are C

The corresponding conditional form can be written as

'If All A are B and All B are C, then all A are C' (1)

Let the first premise be symbolised as P1 and the second premise as P2. The conclusion is symbolised as C. Then the (1) can be symbolised as

$(p1 \cdot p2) \supset C$ (2)

As the given argument (1) is a valid one, its conclusion must be true whenever its premises are true. That means whenever P1 and P2 are true, C must be true. Hence, the symbolisation (2) which states that P1 and P2 implies C must be a tautology. A propositional form is said to be a tautology when it has only a true substitution instance. There are two conditions to be satisfied for C.P to be used.

a. Conclusion must be a conditional proposition

b. It should be possible to deduce a conditional proposition from a conjunction of premises by a sequence of elementary valid arguments, that is, by the instrumentation of the relevant rules of inference.

To every argument, there corresponds a conditional statement whose antecedent is the conjunction of the premises of the argument and whose consequent is the conclusion of the same argument. It was already noted that an argument is valid if and only if its corresponding conditional is a tautology. If an argument has a conditional statement for its conclusion, which may be symbolised as $A \supset C$, then if we symbolise the conjunction of the premises P1 and P2 as P, the argument is only valid if and only if it is conditional.

$P \supset (A \supset C)$ (3)

is a tautology. If we can deduce the conclusion A⊃ C by a sequence of elementary valid arguments from the premises conjoined in P, thereby proving the argument to be valid, then the associated conditional (3) must be a tautology. By the principle of exportation, (3) is logically equivalent to

(P•A) ⊃ C ------- (4)

However, (4) is conditional and associated with a somewhat different argument. The second argument has all of the first premises of the first argument and an additional premise as its premise. This additional premise is the antecedent of the conclusion of the first argument. The conclusion of the second argument is the consequent of the conclusion of the first argument.

If it is deduced that the conclusion of the second argument, C, from the premises conjoined in P•A by a sequence of elementary valid arguments, it can be proved that its associated conditional statement (4) is a tautology.

But since (3) and (4) are logically equivalent, this fact proves that (3) is also a tautology, from which it follows that the original argument with one less premise and the conditional conclusion A ⊃ C is a valid one.

The rule of Conditional Proof permits us to infer the validity of any argument in the following way. The argument.

P

∴ A ⊃ C

may be written as follows by using the method of CP

P

A /∴ C

For any argument whose conclusion is a conditional statement, the proof of its validity can be constructed by assuming the antecedent of its conclusion as an additional premise and then deducing the consequent of its conclusion by using the rules of inference and the rules of replacement.

A conditional proof of validity for the argument

(A v B) ⊃ (C•D)

(D v E) ⊃ F

∴ A ⊃ F

written as

1. (A v B) ⊃ (C•D)
2. (D v E) ⊃ F /∴ A⊃F
3. A /∴ F (C.P)
4. A v B 3. Add.
5. C • D 1,4 M. P
6. D • C 5, Com.
7. D 6, Simp.
8. D v E 7, Add.
9. F 2,8 MP

- Conditional proofs involve proving the consequent of a conditional statement given its antecedent

3.2.3 Strengthened Rule of Conditional Proof

Concerning conditional proof, there is another method called the Strengthened Rule of Conditional Proof. In this method, the antecedent of the conclusion doesn't need to be necessarily assumed. An initial assumption is made, which can either be true or false, but the conclusion must be considered true. The assumed component of the conditional statement can be the antecedent or consequent of the statement considered. This method is called a strengthened rule of conditional proof because it allows multiple assumptions.

Let us consider,

(P • Q) ⊃ R

Assume that P • Q is true. From this assumption, we need to show that R is true.

Now, using the assumption, we take another premise, P ⊃ R.

It is known that if P is true, then R must be true so we can directly derive R.

Since we assumed P to be true, we can use P ⊃ R to conclude R must be true, hence proving

(P • Q) ⊃ R to be true.

The strengthened rule of C.P allows one to assume more than one condition. That is, by assuming P and Q and using known premises, we derived R, proving the validity of (P • Q) ⊃ R.

Another feature of this method is the limit of assumption. The last step is always outside the limits of assumption. If there are two or more assumptions in an argument, then there will be a distinct last step with respect to each assumption. Thus, the range of the application of the condition is defined. In order to easily identify the range of its application, an arrow is used to indicate what is assumed, and with the help of the same arrow, its range is also defined. The application of C.P. is restricted to the space covered by the arrows. All steps outside this arrow are also independent of the condition. Since the conclusion does not depend upon its own antecedent, it has to depend upon the first premise only. In this sense, it is a strengthened condition.

Consider this example,

1. (A v B) ⊃ [(C v D) ⊃ E] /∴ A⊃ [(C • D) ⊃ E]
2. A
3. A v B 2, Add
4. (C v D) ⊃ E 1, 3, M.P.
5. (C•D)
6. C 5, Simp.
7. C v D 6, Add.
8. E 4, 7, M.P.
9. (C • D) ⊃ E 5, 8, C.P.

10. A ⊃ [C • D) ⊃ E] 2, 9, C.P.

3.2.4 Indirect Proof

- Assuming negation of the conclusion

The method of indirect proof is also called the method of proof by *Reductio ad absordum* (Reduces to an Absurdity). This rule has been adopted from elementary geometry. In deriving the theorems, Euclid often begins by assuming the opposite of what he wants to prove. If that assumption leads to a contradiction, then that assumption must be false. Therefore, its negation (that which is to be proved) must be true.

An indirect proof of validity for a given argument is constructed by assuming the negation of its conclusion as an additional premise and then deriving an explicit contradiction from the set of premises. Consider the example:

A ⊃ (B•C)

(B v C) ⊃ E

D v A

∴ E

This can be written as

1. A ⊃ (B•C)	
2. (B v C) ⊃ E	
3. D v A / ∴E	
4. ~ E	I. P
5. ~ (B v D)	2, 4, MT
6. ~B • ~D	5, Dem.
7. ~D • ~ B	6, Comm.
8. ~ D	7, Simp.
9. A	3,8 D.S
10. B • C	1,9 MP
11. B	10, Simp.
12. ~B	6, Simp
13. B • ~B	11, 12 Conj.

- Indirect proof relies on a contradiction to prove the validity of the given argument

Here, it can be seen that when the denial of the conclusion is assumed (along with the truth of the premises), it leads to a contradiction. Since contradiction cannot be accepted, the denial of the conclusion cannot be made. Hence, it is proved that the conclusion of the given argument cannot be false when its premises are true, and hence, the argument is proved to be valid.

Summarized Overview

Though powerful, the rules of inference do not address the varied arguments in constructing the formal proof of validity, so another set of rules is to be instrumented. To widen the range of deduction methods, the rules of inference, the rules of replacement, conditional proof, and indirect proof are employed. This unit details the rules of replacement and how they enrich the rules of inference when constructing formal proof of validity. We also discussed the methods of conditional proof and indirect proof, demonstrating how these rules are to be applied in checking the validity of an argument.

Exercises:

Test the validity of the following arguments by using the Rules of Inference and Rules of Replacement.

1. X ⊃ (Y ⊃ Z)
   X ⊃ (A ⊃ B)
   X • (Y v A)
   ~Z /∴ B

2. C ⊃ (D ⊃ ~ C)
   C ≡ D /∴ ~C v ~D

3. E • (F v G)
   (E • G) ⊃ ~ (H v I)
   ~ (~H v ~I) ⊃ ~ (E • F) /∴ H ≡ I

4. J v (~K v J)
   K v (~J v K) / ∴J ≡ K

5. (E • F) • G
   (F ≡ G) ⊃ (H v I) / ∴ I v H

6. (L v M) v (N • O)

(~ L • O) • ~ (~ L • M) /∴ ~L • N

Test the following arguments using the method of C.P

1. P • (Q ⊃ R) /∴ (P ⊃ Q) ⊃ (P ⊃ R)
2. P ⊃ (Q ⊃ R) /∴ Q ⊃ (P ⊃ R)
3. (A ⊃ B) • (A ⊃ C) /∴ A ⊃ (B v C)
4. (A⊃ B) • (A⊃ C) /∴ A⊃ (B•C)
5. (A ⊃ B) / ∴ (A•C) ⊃ (B•C)

Test the following arguments using the method of I.P

1. A v (B •C)

   A ⊃ C /∴ C

2. (D v E) ⊃ (F ⊃ G)

   (~ G v H) ⊃ (D•F) /∴ G

3. (H ⊃ I) • (J ⊃ K)

   (I v K) ⊃ L

   ~ L / ∴ ~ (H v J)

4. (M v N) ⊃ (O•P)

   (O v Q) ⊃ (~ R • S)

   (R v T) ⊃ (M • N) / ∴ ~ R

5. (V ⊃ ~ W) • (X ⊃Y)

   (~W ⊃ Z) • (Y ⊃ ~ A)

   (Z⊃ ~ B) • (~ A⊃ C)

   V • X / ∴ ~ B • C

Solutions to the Exercises:

Test the validity of the following arguments by using the Rules of Inference and Rules of Replacement.

1. 1 X ⊃ (Y ⊃ Z)

2 X ⊃ (A ⊃ B)	
3 X • (Y v A)	
4 ~Z /∴ B	
5 (X • Y) ⊃ Z	1, Exp.
6 (X • A) ⊃ B	2, Exp.
7 (X • Y) v (X • A)	3, Dist.
8 [(X • Y) ⊃ Z] • [(X • A) ⊃ B]	5,6, Conj.
9 Z v B	8, 7, C.D.
10 B	9, 4, D.S.

2. 1 C ⊃ (D ⊃ ~ C)

2 C ≡ D /∴ ~C v ~D	
3 C ⊃ (~ ~C ⊃ ~D)	1, Exp.
4 C ⊃ (C ⊃ ~D)	3, D.N.
5 (C • C) ⊃ ~D	4, Exp.
6 C ⊃ ~D	5, Taut.
7 ∴ ~C v ~D	6, Impl.

3. 1 E • (F v G)

2 (E • G) ⊃ ~ (H v I)	
3 ~ (~H v ~I) ⊃ ~ (E • F) /∴ H ≡ I	
4 (E • G) ⊃ (~H • ~I)	2, De.M.
5 ~ (H • I) ⊃ ~ (E • F)	3, De.M.
6 (E • F) ⊃ (H • I)	5, Trans.
7 {(E • F) ⊃ (H • I)} • [(E • G) ⊃ (~H • ~I)]	6,4, Conj.
8 (E • F) v (E • G)	1, Dist.
9 (H • I) v (~H • ~I)	7,8, C.D.
10 ∴ H ≡ I	9, Equiv.

4. 1 J v (~K v J)

2 K v (~J v K) / ∴J ≡ K	
3 (~K v J) v J	1, Com.

4 ~K v (J v J)	3, Ass.
5 ~K v J	4, Taut.
6 K ⊃ J	5, Impl.
7 (~J v K) v K	2, Com.
8 ~J v (K v K)	7, Ass.
9 ~J v K	8, Taut.
10 J ⊃ K	9, Impl.
11 (J ⊃ K) • (K ⊃ J)	10, 6, Conj.
12 ∴ J ≡ K	11, Equi.

5.

1 (E • F) • G	
2 (F ≡ G) ⊃ (H v I) / ∴ I v H	
3 E • (F • G)	1, Ass.
4 (F • G) • E	3, Com.
5 (F • G)	4, Simp.
6 (F • G) v (~ F • ~ G)	5, Add.
7 F ≡ G	6, Equiv.
8 H v I	2, 7, M.P.
9 ∴ I v H	8, Com.

6.

1 (L v M) v (N • O)	
2 (~ L • O) • ~ (~ L • M) /∴ ~L • N	
3 ~ L • [O • ~ (~ L • M)]	2,Ass.
4 ~ L	3, Simp.
5 L v [(M v (N • O)]	1, Ass.
6 M v (N • O)	5,4, D.S.
7 ~ (~ L • M) • (~ L • O)	2, Com.

8 ~ (~ L • M) 7. Simp

9 ~~L v ~ M 8, De. M.

10. L v ~ M 9, D.N.

11 ~ M v L 10, Com.

12. ~ M 11, Simp

13. N • O 6, 12, D.S.

14 N 13, Simp.

15 ~ L • N 4,14, Conj.

Test the following arguments using the method of C.P

1. 1. P • (Q ⊃ R) /∴ (P ⊃ Q) ⊃ (P ⊃ R)

2. P ⊃ Q /∴ P ⊃ R C.P.

3. P /∴ R C.P.

4. Q 2,3, MP

5. (Q ⊃ R) • P 1, Com.

6. Q ⊃ R 5, Simp

7. R 4, 6, M.P.

2. 1. P ⊃ (Q ⊃ R) /∴ Q ⊃ (P ⊃ R)

2. Q /∴ P ⊃ R C.P.

3. P /∴ R C.P.

4. Q ⊃ R 1, 3, M.P.

5. R 4, 2, M.P.

3. 1. (A ⊃ B) • (A ⊃ C) /∴ A ⊃ (B v C)

2. A /∴ B v C C.P.

3. A ⊃ B 1, Simp.

4. B 3, 2, M.P.

5. B v C 4, Add.

4. 1. (A⊃ B) • (A⊃ C) / ∴ A⊃ (B•C)

2. A / ∴ B • C	C.P.
3. A ⊃ B	1, Simp.
4. B	3, 2, M.P.
5. (A⊃ C) • (A⊃ B)	1, Com.
6. A ⊃ C	5, Simp.
7. C	6, 2, M.P.
8. B • C	4, 7, Conj.

5. 1. A ⊃ B / ∴ (A • C) ⊃ (B • C)

2. A • C / ∴ B • C	C.P.
3. A	2, Simp.
4. B	1, 3, M.P.
5. C • A	2, Com.
6. C	5, Simp.
7. B • C	4, 6, Conj.

Test the following arguments using the method of I.P

1. 1. A v (B • C)

2. A ⊃ C / ∴ C

3. ~ C	I.P.
4. ~A	2, 3, M.T.
5. B • C	1, 4, D.S.
6. C • B	5, Com.
7. C	6, Simp.
8. C • ~ C	7, 3, Conj

Here, we reach a contradiction; therefore, the argument is valid.

2. (D v E) ⊃ (F ⊃ G)

2. (~ G v H) ⊃ (D • F) / ∴ G

3. ~ G	I.P.
4. ~ G v H	3, Add.
5. D • F	2, 4, M.P.
6. D	5, Simp.
7. D v E	6, Add.
8. F⊃ G	1, 7, M.P.
9. ~ F	8, 3, M.T.
10. F • D	5, Com.
11. F	10, Simp.
12. F • ~ F	11, 9, Conj.

Here, we reach a contradiction; therefore, the argument is valid.

3. 1. (H ⊃ I)•(J ⊃ K)

2. (I v K) ⊃ L

3. ~ L / ∴ ~ (H v J)

4. H v J	I.P.
5. I v K	1, 4, C.D.
6. L	2, 5, M.P.
7. L • ~ L	6, 3, Conj

Here, we reach a contradiction; therefore, the argument is valid.

4. 1. (M v N) ⊃ (O • P)

2. (O v Q) ⊃ (~ R • S)

3. (R v T) ⊃ (M • N) / ∴ ~ R

4. R	I.P.
5. R v T	4, Add.
6. M • N	3, 5, M.P.
7. M	6, Simp.
8. M v N	7, Add.
9. O • P	1, 8, M.P.
10. O	9, Simp.
11. O v Q	10, Add.
12. ~ R • S	2, 11, M.P.
13. ~ R	12, Simp.
14. R • ~ R	4, 13, Conj.

Here, we reach a contradiction; therefore, the argument is valid.

5.

1. (V ⊃ ~ W) • (X ⊃Y)	
2. (~W ⊃ Z) • (Y ⊃ ~ A)	
3. (Z ⊃ ~ B) • (~ A⊃ C)	
4. V • X / ∴ ~ B • C	
5. ~ (~ B • C)	I.P.
6. ~~B v ~C	5, De.M.
7. ~ Z v ~~A	3, 6, D.D.
8. ~~W v ~ Y	2, 7, D.D.
9. ~V v ~X	1, 8, D.D.
10. (V • X) • (~ V v ~ X)	4, 9, Conj.
11. (V • X) • ~ (V • X)	10, De.M.

Here, we reach a contradiction; therefore, the argument is valid.

Self-Assessment

1. Discuss the importance of the rule of replacement. Provide an example of double negation and De Morgan's theorem.
2. What are the basic differences between indirect proof and conditional proof?
3. How does one strengthen a conditional proof?

Assignments

1. Test the validity of the following argument using Conditional Proof and Indirect Proof

   1. (D • E) ⊃ F,

   2. (D ⊃ F) ⊃ G / ∴ E ⊃ G

2. A v (B . C)

   A ɔ C

   ·: C

3. (E v F) ɔ G

   H ɔ (I. J)

   ·: (E ɔ G) . (Hɔ I)

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Space for Learner Engagement for Objective Questions

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UNIT 3
Shorter Truth Table Techniques

Learning Outcomes

After completing this unit, the learner will be able to:

- have an understanding of the shorter truth table techniques
- recognise the advantages of the shorter truth table techniques.
- learn how to employ the shorter truth table techniques for efficient problem-solving systematically.

Background

The truth table method is very burdensome, especially in the case where there are five or more propositions involved. In such cases, 32 rows or more of possible truth value combinations would be needed to check the validity of the argument. The shorter truth table technique helps to check the validity of the argument efficiently without much effort. This unit gives a detailed account of the shorter truth table technique.

Keywords

Shorter truth table technique, Proving invalidity

Discussion

3.3.1 Shorter Truth Table Technique for Proving Invalidity

- The truth table consist of one raw

A truth table can be used to prove the invalidity of a certain argument by demonstrating its invalidity in its specific form. If at least one row in the truth table has truth values set to the statement variables in such a way that it makes the premises true and the conclusion false, then the argument is invalid. Our process of demonstrating invalidity will be shortened if we can create such a truth value assignment without building the complete truth table.

Consider the example,

If the Senator votes against this bill, then he is against penalties against tax evaders.

If the Senator is a tax evader himself, then he is opposed to more severe penalties against tax evaders. Therefore, if the Senator votes against this bill, he is a tax evader himself.

This can be symbolised as

$V \supset O$

$H \supset O$

$\therefore V \supset H$

Instead of constructing a truth table for this argument, we can prove its invalidity by making an assignment of truth values to the component statements V, O, and H, which will make the premises true and the conclusion false. It can be seen that the conclusion can be made false by assigning the truth value T to V and the truth value F to H. Further, both premises can be made true by assigning the truth value T to O.

- This method of proving invalidity is closely related to the truth table method

This method of proving invalidity is closely related to the truth table method. In effect, making the indicated truth value assignment amounts to isolating the row of the complete truth table in which the invalidity of the argument is established.

Compared to drafting an entire truth table, this new technique

V	O	H	V ⊃ O	H ⊃ O	V ⊃ H
T	T	F	T	T	F

for demonstrating invalidity in a single row is shorter. For more complex arguments, the amount of time and work saved is exponential.

3.3.2 Shorter Truth Table Technique to Test the Validity

The Shorter truth table method can also be used to establish the validity of an argument. While used to check the validity, this method employs the *Reductio Ad Absurdum* Method. That is, we start by assuming that:

(a) All premises are necessarily true. When the premises are truth-functionally compound, the truth values of components should be such that the compound proposition is necessarily true.

(b) The conclusion is necessarily taken to be false. When the conclusion is truth-functionally compound, the truth values of components should be such that the conclusion is necessarily false.

So, to prove the validity of an argument, the attempt to prove the invalidity of the argument must lead to a contradiction. That is to say while assigning the truth values to make the premises true and the conclusion false, if any component takes the values 'T' and 'F' simultaneously, then there is a contradiction. Therefore, the assumption that the argument is invalid is false. Hence, it must be valid. It is important to note that once a certain truth value is assigned to a component, it becomes a permanent fixture of that component throughout the argument.

Consider the statement below to prove the validity of the argument,

(A v B) ⊃ (C • D)

(D v E) ⊃ G

∴ A ⊃ G

1. We start by assigning T to each premise and F to the conclusion.
2. Assigning F to the conclusion requires that 'T' be assigned to A and 'F' be assigned to G.
3. Since the 'T' is assigned to A, the antecedent of the first premise is True.
4. The premise has been assumed to be true, so the consequent must also be true. So, T is assigned to both C and D.
5. Since 'T' is assigned to D, the antecedent of the second premise is that the second premise is true.
6. The second premise has been assigned 'T', so its consequent must also be true.
7. So, T must be assigned to G.
8. However, we have already assigned F to G in step (2) to make the conclusion false.
9. Hence, the argument is invalid only if the statement G is both true and false simultaneously, which is impossible.
10. Since the argument cannot be proven invalid, it is a valid argument.

This method of proving the validity of an argument is a version of the reductio ad absurdum technique that uses truth value assignments rather than Rules of Inference.

3.3.3 Classification of Statements

We have seen that the Shorter Truth Table Technique can be used to prove the validity and invalidity of an argument. This method can easily be extended to the classification of statements as tautologous, contradictory or contingent.

- Tautologies contradictory

This is done, firstly, by attempting to give truth values to the component statements to make the given statement false. If we are not able to assign the truth values consistently to make the statement false (that we need to assign both T and F to the same statement somewhere), the given statement is a tautologous one.

Further, we can attempt to give truth values to the component statements to make the given statement true. If we are not able to assign the truth values consistently to make the statement true (that we need to assign both T and F to the same statement somewhere), the given statement is a contradictory one.

For example, to confirm that Pierce's Law [(p ⊃ q) ⊃ p] ⊃ p is a tautology,

- Assign a truth value F to the whole statement.
- This requires assigning T to its antecedent [(p ⊃ q) ⊃ p] and F to its consequent p
- For the conditional [(p ⊃ q) ⊃ p] to be true while its consequent p is false, its antecedent (p ⊃ q) must be assigned the truth value F.
- For the conditional p ⊃ q to be false, its antecedent p must be assigned T, and its consequent q be assigned F.
- However, we already assigned F to p, so assuming Peirce's Law false leads to a contradiction, which proves it to be a tautology.

When truth values are assigned consistently to its components on the assumption that it is false, then the expression in question is not a tautology but is either contradictory or contingent.

As seen before, if this attempt to assign truth values to make the statement false leads to a contradiction, the expression cannot possibly be true and must be a contradiction.

But if truth values can be assigned to make it true and other truth values can make it false, then it is contingent.

Summarized Overview

The *reductio ad absurdum* method of assigning truth values is the quickest and easiest method of testing arguments and classifying statements. However, it is more efficiently used in some cases than others. In the case of disjunction and conjunction, if F is assigned, it should be assigned to both the disjuncts, and T should be assigned to both the conjuncts, respectively. This assignment does not determine the truth and falsity of the simple components of the argument. Despite these complications, the *reductio ad absurdum* is the superior method in most of the cases. In this unit, we discussed the shorter truth table methods, where the validity or invalidity of the argument can be proved without tedious steps or by drawing a complete truth table. The unit also demonstrated the methods to classify statements.

Exercise:

1. Prove the validity or invalidity of the following arguments by using the method of Reductio Ad Absurdum.

1. P ⊃ (Q v R)

~ R / ∴ ~ P

2. (~ W •X)

X ⊃ (Y •Z)

~ Y / ∴ W

Answers:

Prove the validity or invalidity of the following arguments by using the method of Reductio Ad Absurdum.

1. P ⊃ (Q v R)

~ R / ∴ ~ P

1. We start by assuming that the premises are true while the conclusion is false. So, in the first premise, P must be taken as true since the conclusion ~ P is taken as false.
2. To make P ⊃ (Q v R) true, as P is true, Q v R must also be true
3. Q v R can be assigned as true if either Q or R is true.
4. However, in the second premise, ~ R is given to be true, which means R is false.

5. Now, to make Q v R in 3. true, since R is false, Q must be true.

6. It is seen that we can consistently assign truth values T to P and Q and F to R to make the premises true and the conclusion false. This shows that the argument is invalid.

2. ~ W • X

   X ⊃ (Y •Z)

   ~ Y / ∴ W

   1. We start with an assumption that the premises are true and the conclusion is false. If W is false, then we can know that ~W must be true.

   2. The first premise is taken to be true. Hence, X must be true as ~ W is already established as true since we assumed the conclusion to be false.

   3. Since X is true, the consequent of the second premise, Y • Z, should also be true. If so, both Y and Z must be true.

   4. However, it is given in the third premise that ~ Y is true, implying that Y is false. This is a contradiction, which assumes Y to be true in 3. and false in 4. Hence, the argument is valid.

Self-Assessment

1. Write three complex symbolic arguments involving multiple propositions. Use the shorter truth table method to determine their validity or invalidity.
2. Choose two arguments: one simple (with up to 3 propositions) and one complex (with 5 or more propositions). Solve them using both the traditional and shorter truth table methods.

Assignments

1. Prove the validity or invalidity of the following arguments by using the method of Reductio Ad Absurdum.

   1. A ⊃ (B v C)

      ~ B

      ~C / ∴ ~ A

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BLOCK 4
Quantification

UNIT 1

Quantification

Learning Outcomes

By the completion of this unit, the learner will be able to:

- explain the importance and significance of quantifiers
- define and distinguish the universal and existential quantification
- differentiate between singular propositions and general propositions
- convert the propositions into logical statements using appropriate quantifiers
- use variables, predicates, and logical connectives to construct symbolic representations of complex propositions

Background

Logicians have long been fascinated by the certainty of mathematics and have been involved in the idea of bringing this certitude to philosophy. The aspiration of having certainty in language brought about the advancement of logic. This journey of expressing the underlying thought process of a decision through symbols gave way to the advancement of symbolic logic. It is evident to us how logicians have remained keen in dealing with arguments made up of propositions that had connectives like 'and', 'if... then', etc. This inquiry into the underlying structure of an argument was made to understand the validity and invalidity of the argument. Different methods of deduction are employed to prove the validity of a given argument. For this, we use certain elementary valid arguments that can be understood as valid. The methods of deduction gave us a set of twenty rules which aid us in proving the validity and invalidity of an argument made of compound statements. What if we want to test the validity of these elementary arguments? How do we prove that a simple categorical statement like 'All men are mortal' that bases other categorical statements of an argument is valid? This calls for quantification.

The concept of quantification emerged during the late 19th and early 20th century. The idea of quantification was first introduced by Gottlob Frege in his work "*Begriffsschrift*" (concept- Script) in 1879. Frege's contribution laid the foundation stone for the predicate logic that traverses beyond the limitations of Aristotelian syllogism and allows the language to confront arguments that are not compound. This confrontation enabled the analysis of philosophical arguments with greater precision. This was later enriched by logicians and mathematicians, including Bertrand Russell and A.N. Whitehead.

Keywords

Quantification, Existential quantifier, Universal quantifier

Discussion

4.1.1 Quantification

- The limitation of rules of inferance and replacement

Let us recall what we have discussed in the previous block. There, we tested the validity of arguments; to be more specific, we only analysed the arguments whose validity depends on how the simple statements are truth-functionally combined into compound statements. We used the elementary valid argument forms, rules of replacement, conditional proof, etc., to draw inferences that analysed the validity and invalidity of the arguments possible. However, more arguments have simple component statements, which cannot be critically examined by the rules of inference and replacement as they have no provision to look into the interrelationship between the elements of the propositions used. Let's consider the classical example,

All men are mortal

Socrates is a man

Therefore, Socrates is mortal

This argument is definitely valid. There is no doubt to this fact, but using the rules that we have learnt in the preceding units, we may symbolise it as,

A

H

∴ M

By using the methods learned previously, it may appear that it is not valid. The difficulty of finding the validity of this argument arises because the inner logical structure that determines the validity of this argument cannot be analysed by the rules that we have learnt so far. Hence, the above statements can only be symbolised as such, which shall appear blunt. The propositions in this valid argument are not compound and the methods that we have developed deal with compound statements, which are not enough to accommodate simple propositions like the example given above. So, a new method should be employed to symbolise the simple propositions So that their inner logical structure can be expressed by symbolisation. The theory of Quantification provides methods for this symbolisation.

- The theory of Quantification can represent the inner logical structure of the argument

Quantification enables us to convert the simple propositions in the premises into compound propositions without altering the meaning. After this conversion, deduction methods, like Rules of Inference, Rules of Replacement, Conditional Proof, etc., are used to prove validity or invalidity. The compound statement arrived as the conclusion, then, may be converted back to the simple statement that was initially the conclusion.

- Quantification enables us to convert simple propositions into compound propositions without altering the meaning.

The methods of deduction remain fundamental, as adding a quantifier does not interfere with the rules that we discussed earlier. Quantification merely uses some additional symbols to represent the inner structure of the propositions; that is, the relation of the subject and predicate of the simple propositions may be made accessible by quantifiers. Quantification, in short, can be said to be the method for describing and symbolising simple statements in a compound form by expressing the inner logical structure without altering the meaning of the simple argument.

- Method of describing and symbolising simple statements in a compound form

4.1.2 Singular Propositions and their Symbolisation

Let's think back and try to remember what a simple proposition is. A proposition is a declarative statement that has a truth value. The statement is either true or false, and they cannot be both true and false at the same time. A simple proposition has a subject term and a predicate term. In the above example, the second premise of the argument is a singular proposition. It says that the individual Socrates has the attribute of being human. In the proposition 'Socrates is human', 'Socrates' is the

- Proposition is a declarative statement that has a truth value

- Attribute in the predicate is ascribed to a specific subject in Affirmative Singular Propositions

subject term, and 'human' is the predicate term, which is an affirmative singular proposition. The affirmative singular proposition is a proposition in which the attribute in the predicate term is ascribed to the subject term, where the subject term indicates a particular individual, and the predicate term designates some attribute that the individual is said to have.

- Different combinations are possible between subject and predicate terms

The same subject term can be in combination with different predicate terms. For instance, the subject term 'Socrates', when combined with the predicate 'wise', can be formulated as 'Socrates is wise'. Similarly, we can find propositions like, 'Socrates is a scholar', 'Socrates is handsome' etc. We can also have combinations of different subject terms while the predicate term 'human' remains the same. For example, 'Aristotle is human', 'India is human', and 'Palakkad is human'. In the statements here, except for the first one, the other two are false.

- Quantification applies to individuals and diverse predicate types

In quantification, individuals are not only people but things such as animals, cities, nations, etc., of which attributes can be predicated. Attributes do not have to be adjectives such as 'mortal' or 'wise' but can also be nouns. The distinction between adjective and noun is not important in the context of formal logic, and hence, there is no difference between 'Socrates is mortal' and 'Socrates is a mortal' for us. Similarly, predicates can also be verbs.

4.1.2.1 Symbolisation

- Lower case letters for subjects, uppercase for predicates

The first step in symbolisation is distinguishing the subject and predicate teams between the individuals and the attributes they may be said to have. There are two different symbols for referring to individuals and attributes. To denote individuals, we use English alphabets from *a* to *w* in lowercase. These symbols, through *a* to *w,* are ordinarily used in accordance with the first letter of the individual's name. These symbols denoting the individuals are called 'individual constants' for convenience. In the case of Socrates, '*s*' is used, and '*a*' is used to denote Aristotle. So, the subject term is usually denoted by these lowercase alphabets. Similarly, capital letters are employed to symbolise attributes, that is, the predicate term. For example, 'human' is represented by '*H'*, 'Wise' is represented by '*W'* and so on.

Singular propositions can be symbolised by writing an attribute symbol to the left of an individual Symbol. Here, through symbols, we express that the individual named in the proposition has the attribute specified. Thus, the proposition 'Socrates is human' can be symbolised as *'Hs'*. It can be seen that the symbolisation begins with the attribute symbol, H and is followed by a symbol representing an individual. This pattern observed can be symbolised as *Hx*. *Hx* is used to symbolise the general pattern found in all of the singular propositions that attribute being human to the individuals in the proposition. The letter *'x'* in the representation is termed as an individual variable.

- Place attribute symbol before individual symbol in singular proposition

An individual variable *'x'* is simply a place marker that can be replaced with various individual constants representing different individuals. The letters *'x', 'y' and 'z'* serve as variables, while letters only *a* through *w* are generally used as individual constants. It is customary to represent the first individual variable using the letter *'x'*, the second using *'y'* and the third using the letter *'z'*. The symbol *Hx* is known as a propositional function. A propositional function contains an individual variable, and when an individual constant is substituted for the individual variable, it becomes a statement. So, it can be said that any singular proposition is a substitution instance of a propositional function. Some individual constant is substituted for the individual variable in that propositional function of that statement. This process of deriving substitution instances from a propositional function is known as instantiation.

- Individual variables are placeholders; substitution yield statements

A propositional function has true substitution instances and false substitution instances. That is, if *H* symbolises human, *s* symbolises Socrates, and *p* symbolises Palakkad, then *Hs* (Socrates is human) is true, and *Hp* (Palakkad is human) is false. After the substitution is made, the truth and falsity of the proposition can be analysed. But, before this substitution is made, there is only the propositional function, which is neither true nor false. The propositional function that has some true and some false substitution instances is termed a simple predicate. In the examples given above, the predicate 'human' is attributed to both 'Palakkad' and 'Socrates', of which 'Palakkad' is not human. So, the predicate Human can have true or false substitution instances.

- Propositional function's truth and falsity depends on substitution instances

4.1.3 General Propositions and Their Symbolization:

We have already discussed that a singular proposition affirms that the given predicate is designated with the in-

- General Proposition assert attributes for multiple individuals

dividual thing. What if we want to assert the attribute in question to more than one individual? If we are to use 'Everything is mortal' or 'Something is beautiful'. These expressions have the predicate term referring to more than one individual thing. These propositions are termed as General Propositions.

4.1.3.1 Symbolization

Let us consider the general proposition, "Everything is mortal". This proposition can be expressed as "All things are mortal". This proposition may be expressed as:

Given any individual thing, whatever, it is mortal.

All things are mortal
↓
Given any individual thing, whatever, it is mortal
↓
Given any x, x is mortal
↓
Given any x, Mx
↓
(x)Mx

It can be seen in the above formulation that there is no logical inconsistency, and the word 'it' is a pronoun that refers back to the word 'thing'. We have already seen that the letter '*x*' can be used as the individual variable. So we can substitute *x* in the place of 'it' and ' thing', and we can rewrite the above step as

Given any x, x is mortal.

Subsequently, employing the symbolisation of the singular proposition, it can be written as

Given any x, Mx.

It is already known that *Mx* is a propositional function. So, the above formulation has an expression that has *Mx,* making it a proposition. The phrase "Given any x" can be symbolised by *"(x)"*, which is called the Universal Quantifier. Hence, the general proposition "Everything is mortal" can be symbolised as:

(x)Mx

It can be understood here that the propositional functions may be converted into a proposition not only by instantiation but also by generalisation or quantification.

- Universal quantification is symbolised as *(x)Mx* and indicates that a predicate applies to all individuals

Now, let us consider the general proposition "Something is beautiful". Since the technical meaning of 'some' in logic is 'at least one', this proposition can be expressed as:

Something is beautiful

↓

There is at least one thing that is beautiful

↓

There is at least one x such that x is beautiful

↓

There is at least one x such that Bx

↓

$(\exists x)Bx$

There is at least one thing that is beautiful.

In this formulation, the word "that" and "thing" refers to an individual thing, and hence, using the individual variable *x,* the proposition can be rewritten as,

There is at least one x such that x is beautiful.

The sentence now formulated can be written as,

There is at least one x such that Bx.

Bx here is a propositional function, and the phrase 'there is at least one *x* such that' can be symbolised as '$\exists x$'; the symbol '$\exists x$' is called the existential quantifier. So, the proposition everything is beautiful can be symbolised as,

$$(\exists x)Bx$$

which says something is beautiful.

- Existential quantification is symbolised as $(\exists x)Bx$ and indicates that a predicate applies to atleast one individual

Hence, it is seen that propositions are derived from propositional functions either by instantiation, that is, by substituting an individual constant for its individual variable, or by generalisation, that is, by placing a universal or existential quantifier before the propositional function.

Relation between universal and existential quantification:

The universal quantification of a propositional function $(x)Mx$ is true if and only if all the substitution instances of the function are true. While,

The existential quantification of a propositional function, $(\exists x)Mx$, is true if and only if its propositional function has at least one true substitution instance.

Now, let us assume that there exists at least one individual for whom the predicate may or may not be true.

- If $(x)Mx$ is true, then $(\exists x)Mx$ must also be true

Under this assumption, it can be said with certainty that if the universal quantification of a propositional function is true, then its existential quantification must also be true. That is, if every *x* is M, then if there exists at least one thing, that thing is M.

- The symbolisation of negative propositions

It is already known that the propositions are not just affirmative. There would be a substitution instance where one may deny the mortality of Socrates and say that 'Socrates is not mortal', which can be symbolised as *~Ms*. The propositional function of the substitution is *~Mx*.

Let us consider the general proposition,

Nothing is perfect,

which can be paraphrased as,

Everything is imperfect.

which may be written as,

Given any individual thing, whatever, it is not perfect.

which may be rewritten as,

Given any x, x is not perfect.

which can be symbolised as,

(x)~Px. where P symbolises perfect.

Similarly, the proposition "Something is not Mortal' can be represented as *(Ǝx)~Mx.*

The following are the series of relations between universal and existential quantification.

1. The universal general proposition "Everything is mortal" is denied by the existential general proposition "Something is not mortal.". This can be said that *(x)Mx* is denied by *(Ǝx)~Mx*. Since both propositions that are in denial of each other are biconditional, they can be made logically true by prefacing one with a negative symbol.

$$\sim(x)Mx \equiv (\exists x)\sim Mx$$

2. The universal general proposition "Everything is mortal" expresses exactly as "There is not even one thing that is not mortal". This can be formulated as:

$$(x)Mx \equiv \sim(\exists x\ )\sim Mx$$

- Universal and existential propositions can be related through negation and biconditional statements

3. The universal general proposition, “Nothing is mortal,” is denied by the existential general proposition, “Something is mortal”. This can be Symbolised as:

$$\sim(x)\sim Mx \equiv (\exists x)Mx$$

4. “ Everything is not mortal” is logically equivalent to the expression “There is nothing that is mortal”. This may be expressed as:

$$(x)\sim Mx \equiv \sim(\exists x)Mx$$

By replacing the predicate M with the Symbol Φ, the Greek letter phi, the above biconditionals can be rewritten as:

$$[(\exists x)\sim\Phi x] \equiv [\sim(x)\Phi x]$$

$$[(x)\Phi x] \equiv [\sim(\exists x\ \ \sim\Phi x]$$

$$[(\exists x)\Phi x] \equiv\ [\sim(x\ \sim\Phi x]$$

$$[(x)\sim\Phi x] \equiv [\sim(\exists x)\Phi x]$$

- The Graphical representation of the connections between universal and existential quantification

The general connections between universal and existential quantification can be depicted graphically with the help of the traditional square of opposition. From the square of opposition given, we can derive the following relations. This assumes the existence of at least one individual in the class ‘x’.

1. The two top propositions are contraries; that is, they may both be false together but cannot both be true together.
2. The two bottom propositions are subcontraries; that is, they may both be true together but cannot both be false together.
3. Propositions that are at opposite ends of the diagonals are contradictories, of which one must be true, and the other must be false.
4. On each side of the square, the truth of the lower proposition is implied by the truth of the proposition directly above it.

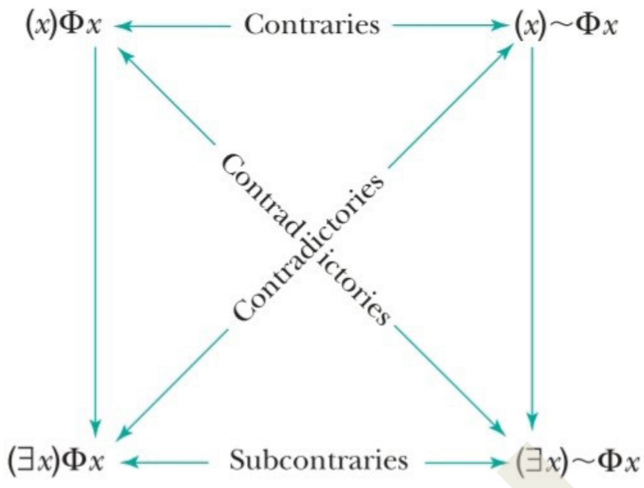

Fig 4.1.1 (Copi, Cohen, Figure 1, p. 441)

Summarized Overview

In this unit, we discussed what quantification is and how it enhances our analytical machinery. We have also learnt what an affirmative singular proposition is and how the propositions can be symbolised. The unit also explained the singular and general propositions, detailing how these propositions are symbolised. The unit concluded with a discussion on the relation between the universal and existential quantifiers with the help of the square of opposition.

Exercises:

1. Briefly explain the following terms in one or two sentences.

    a. Quantification

    b. Individual constant

    c. Individual variable

    d. Universal Quantifier

    e. Existential Quantifier

2. Name the process of substituting an individual constant for an individual variable, thereby converts into the proposition.

3. What is Generalization?

4. What do you mean by propositional function?

5. What is an affirmative singular proposition?

Answers:

1. a) Quantification is a method for describing and symbolising simple statements by reference to their inner logical structure.

   b) Individual constant is a symbol used in logical notation to denote an individual thing. The symbolisation utilises the lowercase from a to w.

   c) An individual variable serves as a placeholder for an individual constant. Usually represented by the letters x and y.

   d) Universal quantifier is the symbol used before a propositional function to assert that the predicate following the symbol is true of everything that is part of that which is asserted through the predicate. This is symbolised as (x).

   e) Existential Quantifier: The symbol used before a propositional function to assert that the function has one or more true substitution instances. This is represented by the symbol Ǝ.

2. Instantiation

3. Generalization in quantification is the process of forming a proposition from a propositional function by placing a universal quantifier or an existential quantifier before it.

4. A propositional function is an expression that contains an individual variable and becomes a statement when that variable is replaced with a particular constant. A propositional function can also become a statement by the process of generalisation.

5. Affirmative singular proposition is that in which a particular individual is asserted with some specified attribute.

Assignments

1. Why do we need the method of Quantification?
2. By using the Square of opposition, discuss the relation between universal and existential quantification.
3. Explain singular propositions and their symbolisation with an example.

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1. Copi, I. M, Cohen, Carl, McMahon K, (2014), Introduction to Logic 14th Edition, Pearson.
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Space for Learner Engagement for Objective Questions

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UNIT 2
Symbolisation of Categorical Propositions

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- properly assign variable and logical expressions to paraphrase the proposition.
- translate categorical propositions into symbolic form using appropriate quantifiers
- understand how to express general propositions using quantifiers, going beyond simple subject- predicate forms

Background

In Logic, a categorical proposition can be said to be a proposition that affirms or denies the relation between two classes or categories of objects. That is, a categorical proposition says whether and to what extent (partially or fully) one class overlaps with the other class. The propositions like, 'All dogs are mammals' or "Some cats are black" are some examples of categorical propositions. Traditional logic emphasised four types of general propositions, which were given by Aristotle. The four standard general propositions are as follows:

All humans are mortal (Universal affirmative), which is the 'A' proposition.

No humans are mortal (Universal negative), which is the 'E' proposition.

Some humans are mortal (Particular affirmative), which is the 'I' proposition.

Some humans are not mortal (Particular negative), which is the 'O' proposition.

These propositions are commonly referred to by their letters. The two affirmative propositions, A and I, are derived from the Latin *affirmo* (*AffIrmo)*, meaning "I affirm". Similarly, the two negative prepositions, E and O, are from the Latin word *nego* (***NegO*)**, which means "I deny".

Symbolising these propositions widens the conception of a propositional function. The general propositions are expressed with the help of the Universal and Existential quantifiers. Let us proceed to symbolise the categorical propositions.

Keywords

Quantification of propositions, the square of opposition, Contrary, Sub-contrary, Contradicting

Discussion

4.2.1 Quantification of A type Proposition

- A Proposition is expressed as *(x)(Hx ⊃ Mx)*

Consider the proposition 'All humans are mortal'; the first step in the symbolisation is to paraphrase the proposition as,

Given any individual thing whatever, if it is human, then it is mortal

The pronoun 'it' and the word 'thing' can be replaced by x and can be rewritten as,

Given any x, if x is human, then x is mortal

This can again be rewritten by using the symbol designated for "if….then".

Given any x, x is human ⊃ x is mortal.

Finally, the A proposition can be expressed as

(x)(Hx ⊃ Mx)

In the symbolic translation, the A proposition seems to have a different kind of propositional function than what we have learnt in the previous unit. The expression '*Hx ⊃ Mx*' is a propositional function that has substitution instances of neither affirmative nor negative singular propositions but is a conditional statement that has singular propositions as its antecedent and consequent. The substitution instances of the propositional function '*Hx ⊃ Mx*' are conditional statements. For Example, *Ha ⊃ Ma*, *Hb ⊃ Mb*, etc.

4.2.2 Quantification of the E Proposition

Consider the proposition, "No humans are mortal". This proposition can be paraphrased as:

Given any individual thing, whatever, if it is human, then it is not mortal

This can be written as,

For any given thing x, if x is a human, then x is not mortal.

- E Proposition is expressed as *(x)(Hx ⊃ ~Mx)*

Substituting the horseshoe, the above sentence can be symbolised as

Given any x, x is a human ⊃ x is not mortal

The E proposition can be expressed as:

$$(x)(Hx \supset \sim Mx)$$

This symbolic translation not only expresses the traditional E proposition but also shows the diverse ways of saying the same thing.

4.2.3 Quantification of I Proposition

Consider the I proposition, "Some humans are mortal" This proposition can be paraphrased as

There is at least one thing that is human and mortal

This can be reworded as,

- I Proposition is expressed as *(∃x)(Hx • Mx)*

There is at least one thing such that it is human and it is mortal.

This can be rewritten as,

There is one x such that x is human and x is mortal.

This may be symbolised as,

$$(\exists x)(Hx \cdot Mx)$$

4.2.4 Quantification of O Proposition

Consider the O proposition, "Some humans are not mortal", which may be paraphrased as:

- O Proposition is expressed as $(\exists x)(Hx \bullet \sim Mx)$

There is at least one thing that is human and not mortal

This can be rewritten as,

There is at least one x, such that x is human and x is not mortal.

This can be reworded as,

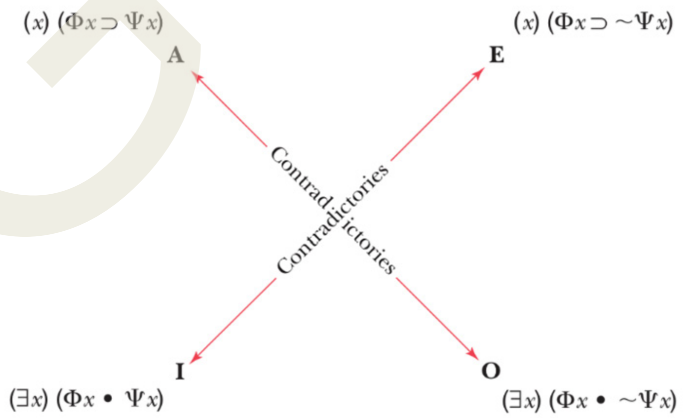
There is at least one x such that x is a human • x is not mortal.

The above statement can be symbolised as

$(\exists x)(Hx \bullet \sim Mx)$

4.2.5 Reworking the Traditional Square of Opposition

In this square of opposition, 'A' and 'O' are contradictories, and 'E' and 'I' are also contradictories. None of the other relationships found in the Aristotelian square of opposition are present in this square of opposition. In this figure, the Greek letters phi (Φ) and psi (Ψ) are used to represent the predicates.



(Copi, Cohen, Figure 2, p. 445)

The new formulation of our universal categorical propositions incorporates interpretation.

When Φx is a propositional function that has no true substitutional instances, then, regardless of what the attribute is symbolised by Ψ, the propositional function 'Φx ⊃ Ψx'

- The denial of contrary relation between A and E propositions

and '(Φx⊃~Ψx)' have only true substitution instances. This is so because all their substitution instances are conditional statements with false antecedents. In such cases, both the A and E propositions that are the universal of these complex propositional functions are true, so the A and E propositions are not contraries.

- The denial of sub-contrary relation between I and O propositions

Also, when Φx is a propositional function that has no true substitutional instances, then regardless of what Ψx might be, the propositional functions '(Φx • Ψx)' and '(Φx • ~Ψx)' have only false substitutional instances, because their substitution instances are conjunctions whose first conjuncts are false. In such cases, the I and O propositions that are the existential quantification of these propositional functions are false. Thus, I and O are not sub-contraries.

Since the A and I propositions are true and the I and O propositions are false, the truth of a universal doesn't imply the truth of the corresponding particular, and they do not hold the relation of implication with each other.

If we are to assume that there is at least one individual, then *(x)(Φx ⊃ Ψx)* does not imply *(∃x)(Φx ⊃ Ψx)*. The latter, however, is not an I proposition as an I proposition is in the form 'some Φs are Ψs is symbolised as ∃x(Φx • Ψx), which asserts that there is at least one thing having the attribute Φ and the attribute Ψ. But the *proposition (∃x)(Φx ⊃ Ψx)* asserts only that there is at least one object that either has the attribute Ψ or does not have the attribute Φ. This is a weaker assertion.

- General proposition extend beyond traditional subject-predicate forms

The four traditional subject-predicate forms, A, E, I, and O, are not the only forms of general propositions. Others involve the quantification of more complicated propositional functions. For Example, 'All members are either parents or teachers," which does not mean the same as "All members are parents or all members are teachers", but they are both symbolised as *(x)[Mx ⊃ (Px v Tx)]*. English has so many irregularities, and hence, translating English sentences into logical notations is complex and often confusing. This can be avoided by carefully analysing and understanding the meaning of the given sentence before rewriting it in terms of propositional functions and quantifiers.

Summarized Overview

In this unit, we have discussed the symbolic representation of categorical propositions using universal and existential quantifiers. The unit explained in detail each type of traditional proposition, namely, A, E, I, and O propositions. A proposition asserts that all instances of substitution possess that which is attributed by the predicate term. The E proposition denies that which is attributed by the predicate term for all the substitution instances. The I proposition affirms that at least one of the substitution instances is true of the predicate term. The O propositions claim that in at least one of the substitution instances, it is not true. The reworking of the traditional square of opposition was also discussed by highlighting that while A and O, E and I propositions are in a contradictory relation like in the traditional square of opposition, the other relation of contraries, sub-contraries, etc., are absent. Why this is so has also been discussed. The unit also underscored the importance of careful analysis to accurately translate the English sentences that do not come under the four traditional propositions into logical notations.

Exercises:

Translate the following sentences into logical notations.

1. Snakes are reptiles (Sx: x is a snake. Rx: x is a reptile)
2. Children are present. (Cx: x is a child. Px: x is present)
3. Only executives have secretaries. (Ex: x is an executive, Sx: x has a secretary)
4. None but the brave deserves the fair. (Bx: x is brave. Dx: x deserves the fair.)
5. Only students who study diligently pass the exam. (Sx: x is a student. Dx: x studies diligently. Px: x passes the exam.)

Answers:

1. The sentence 'Snakes are reptiles' can be paraphrased as, *Given any individual thing, if it is a snake, then it is a reptile.* This can be rewritten as *Given any x, x is a snake ⊃ x is a reptile.* This statement can be symbolised as (x)(Sx ⊃ Rx).
2. The sentence 'Children are present' can be written as *There exists at least one x, such that x is a child and x is present.* This statement can then be symbolised as (∃x)(Cx•Px)
3. 'Only executives have secretaries' can be written as *Given any individual thing x, if x has a secretary, then x is an executive.* The statement can be symbolised as

   (x)(Sx ⊃ Ex)

4. 'None but the brave deserves the fair.' can be written as *Given any individual thing x, if x deserves the fair, then x is brave.* This can be symbolised as (x)(Dx ⊃ Bx).
5. 'Only students who study diligently pass the exam.' can be written as *for any individual thing x; if x passes the exam, then x is a student, and he studies diligently.* This can be symbolised as (x)[(Px ⊃ (Sx • Dx)].

Assignments

1. Translate the sentence 'No one can enter the contest without a valid ID' into logical notation using propositional functions and quantifiers.
2. Why does the modern square of opposition not have any other relationship except for contradiction, as claimed by the traditional square of opposition?
3. Symbolise the A proposition 'All humans are mortal' by using propositional function and quantifier.

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UNIT 3
Preliminary Quantification Rules

Learning Outcomes

Upon the completion of this unit, the learner will be able to:

- state and explain the four preliminary rules of quantification
- employ these rules with the rules of inference to construct the proofs of validity
- translate sentences of natural language using logical notations

Background

In logic, constructing formal proofs of validity is crucial for demonstrating that an argument's conclusion logically follows from its premises, that is, proving the validity of an argument. While many arguments involve compound statements composed of simpler ones connected by logical operators, some arguments depend on the internal structures of these simple, noncompound statements. In the earlier block, we have seen how the validity of arguments made of compound statements is proved. However, the rules we used earlier are not enough to analyse the arguments made of simple non-compound propositions. To address such cases, we must expand our toolkit of inference rules with the four preliminary rules of quantification. This expanded set of rules enables us to delve into the specifics of individual statements, ensuring our logical proofs remain robust and comprehensive. In this unit, we explore the use of these additional rules through examples.

Keywords

Universal Instantiation (UI), Universal Generalization (UG), Existential Generalization (EG), Existential Instantiation (EI)

Discussion

4.3.1 Preliminary Quantification Rules

- Four rules of quantification

To prove the validity of an argument, especially when the argument depends on the specific details inside simple statements, we need to add a few more rules to our toolkit along with the rules of inference, rules of replacement, conditional proof and indirect proof. We need four extra rules of quantification. They are:

- Universal Instantiation (UI)
- Universal Generalization (UG)
- Existential Generalization (EG)
- Existential Instantiation (EI)

4.3.1.1 Universal Instantiation (UI):

- Instance of using the universal instantiation method

Imagine a team of scientists going on an expedition to a remote island; they discover a new species of rare orchids. They were excited about the discovery and began to study the rare species of orchid. To make their study easier, they employed the method of Universal Instantiation. How did that work, one may ask? Using universal instantiation, they inferred that the newly discovered orchid, being an orchid, must have the common features of an orchid. So, they can base their study on the specific nature of orchids rather than needing to start the study all over. This may force you to ask the question, what really is Universal Instantiation?

- Universal instantiation allows inferring specific cases from general triats

The universal quantification of a propositional function is true if and only if all substitution instances of that propositional function are true; this principle can be added to the rules of inference. The Universal Instantiation states that "any substitution instance of a proportional function can be validly inferred from its universal quantification". That is, the scientists understood the features of the newly found species of orchid from the general characteristics of the orchid that have been found and studied. That is, this rule permits substitution instances to be inferred from universal quantifications.

The rule of universal instantiation can be symbolised as

$(x)(\Phi)$

∴ *Φν* (where ν is any individual symbol). The Greek letter *nu* (ν) is used to represent any individual symbol.

With this new rule, the formal proof of validity for the following argument can be formulated.

All humans are mortal

Socrates is human

∴ Socrates is mortal

1. x (Hx ⊃ Mx) (All humans are mortal)
2. Hs (Socrates is human) / ∴ Ms (Socrates is mortal)
3. Hs ⊃ Ms 1, U.I
4. Ms 3, 2 MP

Universal Instantiation is the logical process of arriving at a specific instance, asserting that what is true for all members of a category is true for a particular member of that category.

4.3.1.2 Universal Generalisation (U.G)

- Universal generalisation is the method of inferring a general statement about all the members of a class based on the observation of a particular instance.

Imagine examining a mobile phone to evaluate its overall quality. You might start by inspecting any randomly selected component, like the screen. If you find that this particular part is of high quality, then you will have a positive impression of identical mobile phone models even before examining them. The screen you inspected represents all the individual members of the selected phone model. So, suppose you use the quality of the screen as the criterion for assessing the other phones of the same kind. In that case, it can be said that if this particular part of the selected model is of high quality, then the screen of all the identical phones is of high quality.

If the analogy of the triangle is taken, then the geometer begins a proof by saying, 'Let ABC be any arbitrarily selected triangle', like the 'randomly selected phone'. If it was proved that the triangle ABC has some specified attribute and concludes that all triangles have that attribute, then the small letter 'y' may be used to denote the phrase, 'Let ABC be any arbitrarily selected triangle' or 'randomly selected phone'. In this usage, the expression 'Φy' is a substitution instance of the propositional function 'Φx', and it asserts that any arbitrarily selected individual has the property ‘Φ’.

Clearly, ‘Φy’ follows validly from ‘(x)(Φx) by UI since what is true of all individuals is true of any arbitrarily selected individual. This inference is equally valid in the other direction. Hence, the rule of Universal Generalization can be stated as, ‘what is true of any arbitrarily selected individual must be true of all individuals.’ The rules of inference can be enhanced by adding the principle that the universal quantification of a propositional function can validly be inferred from its substitution instance with respect to the symbol ‘y’. This principle is referred to as the ‘principle of Universal Generalization’. The symbolic expression of UG is

Φy (where ‘y’ denotes any arbitrarily selected individual, and Φy is not within the scope of any assumption containing the special symbol ‘y’)

∴ (x)(Φx)

We can now use the new notation and the additional rule UG to construct a formal proof of validity for the argument,

No mortals are perfect. --------- 1

All humans are mortal. ----------- 2

Therefore, no humans are perfect.

The argument can be symbolised as;

1. (x) (Mx ⊃ ~Px)

2. (x) (Hx ⊃ Mx) / ∴ (x) (Hx ⊃ ~Px)

3. Hy ⊃ My 2, UI

4. My ⊃ ~Py 1, UI

5. Hy ⊃ ~Py 3, 4, H.S

6. (x) (Hx ⊃ ~Px) 5, UG

4.3.1.3 Existential Generalization (E.G)

- Existential generalisation is a logical principle that states that if a predicate is true for at least one specific individual, then it can be said that there exists at least one individual for which the predicate holds true.

Imagine that one is in a garden and spots one red rose among the many flowers. Using the method of existential generalisation, it can be asserted that 'There exists at least one red rose in this garden'. It means that if at least one specific example where a propositional function is true exists, then it can be confidently said that the function is true in some cases in general. The rule says that the existential quantification of a propositional function is true if and only if that propositional function has at least one true substitution instance. So, it can be stated that 'the existential quantification of a propositional function can validly be inferred from any substitutional instance of that propositional function'. Since this rule permits the inference of general propositions that are existentially quantified, it is termed Existential Generalization, which can be symbolised as,

$\Phi \nu$ / $\therefore$ $(\exists x)(\Phi x)$ (where ν is any individual symbol)

4.3.1.4 Existential Instantiation (E.I)

- Existential instantiation is a logical principle that states that from the existential quantification of a propositional function, we can infer the truth of its substitution instance.

The existential quantification of a propositional function asserts that at least one individual exists, the substitution of which for the variable 'x' in that propositional function will yield a true substitution instance of it. Imagine that a detective in a story finds a note saying, 'A witness who saw the crime exists. ' The detective does not know who the witness is, but they called the witness 'Witness W, ' which makes it easier to discuss and infer things related to the witness. Certainly, like the detective, one may not know anything about the substitutional instance. However, one can take any individual constant other than 'y' (for instance, w), which has no prior occurrence in that context, and use it to denote the individual whose existence has been asserted by the existential quantification. The rule is that 'we can infer from the existential quantification of a propositional function their substitution instance of that propositional function with respect to the individual symbol 'w.' The rule can be symbolised as:

$(\exists x)(\Phi x)$ / $\therefore \Phi \nu$ (where ν is an individual constant, other than 'y', that has no prior occurrence in the context)

4.3.2 Formal Proof of Validity

Let us make use of the quantification rules along with other rules and construct a formal proof of the validity of the argument after symbolising the argument,

All dogs are carnivorous. (1)

Some animals are dogs. (2)

Therefore, some animals are carnivorous. (3)

Firstly, the propositions are to be symbolised. Let us assume that the number of steps taken to reach the symbolisation of each proposition be taken as a, b, c, d …. etc

1. All dogs are carnivorous.

a. Given an individual thing, whatever it is, if it is a dog, then it is carnivorous

b. Given any x, if x is a dog, then x is carnivorous.

c. Given any x, x is dog ⊃ x is carnivorous.

d. (x) Dx ⊃ Cx

2. Some animals are dogs.

a. There is at least one thing: an animal and a dog.

b. There is at least one thing such that it is an animal, and it is a dog.

c. There is at least one x such that x is animal and x is dog.

d. There is at least one x such that x is animal • x is dog

e. (∃x)(Ax • Dx)

3. Some animals are carnivorous.

a. There is at least one thing, which is animal and carnivorous.

b. There is at least one thing, such that it is animal and it is carnivorous.

c. There is at least one x, such that x is animal and x is carnivorous.

d. There is at least one x, such that x is animal • x is carnivorous.

e. (∃x)(Ax • Cx)

The following logical notation is derived by symbolising the given argument.

1. (x)(Dx ⊃ Cx)
2. (∃x)(Ax • Dx) / ∴ (∃x)(Ax . Cx)
3. Aw • Dw 2, EI
4. Dw ⊃ Cw 1, UI
5. Dw • Aw 3, Com.
6. Dw5, Simp
7. Cw4, 6, M.P
8. Aw3, Simp
9. Aw • Cw 8,7, Conj.
10. (∃x)(Ax • Cx) 9, EG

Summarized Overview

This unit mainly discussed the four extra rules that are to be added to our toolkit for testing the validity of the argument, along with the rules of inference. Universal Instantiation (UI) infer that if a statement is true for all instances, it is also true for any specific instances. Universal Generalization (UG) says that if something is true for any arbitrarily selected instance, it must be true for all instances. Existential Generalization (EG) states that if a statement is true for at least one specific instance, then it can be inferred that it is true for some cases in general. Existential Instantiation (EI) allows us to introduce a new constant to represent an unknown individual for which a statement is true.

Exercises:

1. What logical principle does Universal Instantiation rely on with respect to the propositional function?
2. Provide the symbolic expression for Universal instantiation.
3. What is Universal Generalization?
4. Explain the principle behind Existential Generalization and provide its symbolic expression.

5. Construct a formal proof of validity for the following arguments:

(x) (Ax ⊃ ~Bx)

(∃x)(Cx • Ax)

∴ (∃x)(Cx • ~Bx)

6. Construct a formal proof of validity for the following argument using the suggested notations.

All mountaineers are neighbourly. Some outlaws are mountaineers. Therefore, some outlaws are neighbourly. (Mx, Nx, Ox)

Answers:

1. Universal Instantiation relies on the principle that the universal quantification of a propositional function is true if and only if all substitution instances of that function are true.

2. The rule of universal instantiation can be symbolised as,

   (x)(Φ)

   ∴Φν (where ν is any individual symbol). The Greek letter nu (ν) is used to represent any individual symbol.

3. Universal Generalization is the principle that if something is true for any arbitrarily selected instance, it must be true for all instances.

4. The Principle of Existential Generalization is that if there exists at least one true substitution instance of a propositional function, we can infer its existential quantification. Existential Generalization can be symbolised as,

   Φν / ∴ (∃x)(Φx) (where ν is any individual symbol)

5. 1. (x) (Ax ⊃ ~Bx)

   2. (∃x)(Cx • Ax) / ∴ (∃x)(Cx • ~Bx)

3. Cw • Aw	2, E.I
4.Aw ⊃ ~ Bw	1, U.I
5. Aw • Cw	3, Comm.
6. Aw	5, Simp

7.~Bw	4,6, M.P
8. Cw	3, Simp.
9. Cw • ~Bw	8,7, Conj.
10. (∃x)(Cx • ~Bx)	9, E.G

6. The proposition, 'All mountaineers are neighbourly,' can be symbolised as,

   (x)(Mx ⊃ Nx)

   Some outlaws are mountaineers, which can be symbolised as (∃x)(Ox • Mx)

   Some outlaws are neighbourly and can be symbolised as (∃x)(Ox • Nx)

   Now, let us move on to construct the formal proof of validity.

   1. (x)(Mx ⊃ Nx)

   2. (∃x)(Ox • Mx) / ∴ (∃x)(Ox • Nx)

3. Ow • Mw	2, EI
4. Mw ⊃ Nw	1, UI
5. Ow	3, Simp
6. Mw • Ow	3, Com.
7. Mw	6, Simp
8. Nw	4, 7, MP
9. Ow • Nw	5,8, Conj
10. (∃x)(Ox • Nx)	9, EG

Assignments

1. Consider the given argument and paraphrase each of the arguments into their logical notation. Then, construct the formal proof of validity for the symbolised argument.

   No athletes are bookworms.

   Carol is a bookworm.

   Therefore, Carol is not an athlete.

2. Describe the four preliminary rules of quantification in detail with examples.

Reference

1. Copi, I. M. (1979). Symbolic Logic fifth edition. PHI Learning Private Limited.
2. Copi, I.M, Cohen, C & McMahnon K, (2014), Introduction to Logic 14th Edition, Pearson

Suggested Reading

1. Klenk, V. (2007). Understanding Symbolic Logic (5th ed.). Prentice Hall.
2. O'Connor, B. (1993). An Introduction to Symbolic Logic. Pearsons

Space for Learner Engagement for Objective Questions

Learners are encouraged to develop objective questions based on the content in the paragraph as a sign of their comprehension of the content. The Learners may reflect on the recap bullets and relate their understanding with the narrative in order to frame objective questions from the given text. The University expects that 1 - 2 questions are developed for each paragraph. The space given below can be used for listing the questions.

Model Question Paper Sets

SREENARAYANAGURU OPEN UNIVERSITY

QP CODE: ……… Reg. No : ………...............
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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - **M23PH05DC WESTERN PHILOSOPHY II** (CBCS - PG)
2023-24 - Admission Onwards

Time: 3 Hours Max Marks: 70

SECTION A

Objective Types Questions; Answer **any Ten** (1x 10=10)

1. Name the philosopher who propounded the Copernican Revolution
2. Enlightenment philosophy propagated that reason has autonomy to control nature and improve human life. True or false?
3. What does Kant mean by a priori elements of knowledge?
4. Did Hegel believe in an organic conception of reality?
5. What is historicism in Philosophy?
6. Is Hegel a philosopher of history?
7. Was Marx a critic of modernity?
8. Did Marx state that the economic foundation of a society conditions its intellectual life?
9. Critical philosophy of Kant is a response to the crisis in the age of enlightenment due to the free play of reason. True or false?
10. Profit is the aim of production in the capitalist mode of production. Who says this?
11. What does ‘base’ and ‘superstructure’ stand for in Marx’s philosophy?
12. Name the three proponents of critical theory.
13. What is the meaning of instrumental reason?
14. What is culture industry in the critical theory?
15. What is the meaning of mass deception?

SECTION B

Very Short Answer Questions; answer **any Five (**2x5=10)

16. What is critical philosophy?
17. Kant has admiration for the physical world as well as the deeper dimensions of humanity. Elaborate his famous statement in this regard.
18. David Hume, an empiricist from Britain, made a striking philosophical argument that shakes the grounds of science. Explain
19. According to Hegel, even if an individual separates himself from other individuals, he cannot isolate himself from and step beyond the spirit of people. He cannot separate himself from the beliefs and values of the society in his times. Discuss.
20. Marx talks about different types of alienation. What are they? Explain
21. Hegel propounds that a man owes to the State for everything he is. He can find his essence in the State only. Therefore, Hegel advocates that there is no higher court of appeal than State. It is the march of God on earth - Explain Hegel's take on the state.
22. What is use-value and exchange-value in Marx's philosophy?
23. Explain the fetishism of commodities in Marx's philosophy
24. Name different types of domination propounded in the culture industry.
25. Adorno attempts a critique of rationality. Explain

SECTION C

Short Answer Questions; answer **any five (**4x5=20)

26. Enlightenment enforces widespread confidence and optimism in the autonomy of reason to control nature and improve human life. Evaluate the statement
27. Marcuse criticized the modern society as one-dimensional. Explain the concept.
28. Explain the intertwined relation of theory and praxis in critical theory.
29. Explain Kant's synthesis of empiricism and rationalism
30. Compare and contrast the dialectical idealism and materialism.
31. "Hegel points out that a nation is the embodiment of the 'spirit of a people'. By 'spirit of a people', he means the language, religion, art, music, poetry, architecture, morality, philosophy, science, and law, of a specific group of people. He considers all these as

forming a totality. None among these can be isolated from the whole. The spirit of a group of people expresses itself through all these elements." Analyze this statement.

32. Discuss how does the capitalist generates profit according to Marx.

33. Hegel viewed that history can be seen as the progress in the consciousness of freedom through his account of the history. Comment.

Section D

Long answer Question; Answer **any three** (10x3=30)

34. Control over material production ensures control over mental production. Debate

35. Theory intertwines with history. Analyze the statement according to the Frankfurt school.

36. What is Hegel's idea of universal spirit? Explain

37. "A society becomes one-dimensional when the thought and behaviour of its members conform to a single pattern. There is neither any instance of dissent from the existing social norms and values nor the demand for an alternative social order." Critically evaluate the one-dimensional society propounded by Marcuse.

38. Explain the Synthetic-Analytic distinction in Kant's philosophy. Also, elaborate the Synthetic Apriori Judgements. What was Kant's aim in bringing all these categories?

39. Give a detailed elucidation of the major philosophical problems of the Frankfurt school.

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - **M23PH05DC WESTERN PHILOSOPHY II** (CBCS - PG)

2023-24 - Admission Onwards

Time: 3 Hours | Max Marks: 70

SECTION A

Objective Types Questions; Answer **any Ten** (1x 10=10)

1. Who made the statement that the mind has increasing admiration and awe for two things – 'the starry heavens above' and 'the moral law within'?
2. What is the focus of the enlightenment philosophy; mind or body?
3. What is the difference between apriori and aposteriori knowledge in Kant?
4. Critical Philosophy takes Philosophy as Critique. Is it true or false?
5. Enlightenment era is the age of criticism for Kant. What is the meaning?
6. What is the meaning of organic conception of reality, according to Hegel?
7. What is the historicism in Hegel's Philosophy?
8. What is the driving force in Hegel's philosophy of history, mind or matter?
9. Hegel's conception of history is about the progress of consciousness. True or false?
10. Marx criticized advanced industrialism of modernity. True or false?
11. Does Marx uphold that the economic foundation of a society conditions its intellectual life?
12. What is the capitalist mode of production according to Marx?
13. Who authored the work *Dialectic of Enlightenment?*
14. What is the meaning of the instrumental reason?
15. Who authored the book *One-dimensional Man: Studies in the Ideology of Advanced Industrial Society*?

SECTION B

Very Short Answer Questions; answer any Five (2x5=10)

16. Explain the notion "Critical philosophy is Philosophy as Critique"

17. Kant faced a difficulty with respect to the development of science on how to explain or justify scientific knowledge. Explain the Hume's attack on science.
18. Kant is not aiming at a critique of books and systems, but a critique of the faculty of knowledge in general. Explain
19. What are the basic tenets of German Idealism?
20. Hegel proposes reality as dynamic and concrete. Explain
21. Oppositions and contradictions are the most crucial elements in Hegel's philosophy. How?
22. How Marx's materialism is different from other materialistic philosophies?
23. Marx reinterprets the history of Hegel as the alienation and self-realisation of man. Comment
24. What is the instrumental reason according to the critical theory?
25. What is hegemony according to Antonio Gramsci?

SECTION C

Short Answer Questions; answer any five (4x5=20)

26. No place for free will and morality in the mechanical world conceived by science according to Kant. Explain the Kantian problem and solution.
27. Kant tried to reconcile the empiricism and rationalism. How did he do that?
28. Explain the historical background to Kant's philosophy of critique of reason.
29. "Every stage in the process contains all the preceding stages and foreshadows all the future ones: the world at every stage is both a product and a prophecy. The lower form is negated in the higher, that is, it is not what it was; but it is also preserved in the higher, it has been carried over and sublated." Explain Frank Thilly's statement about Hegel's conception of reality.
30. There is an inadequacy of deductive method in explaining dynamic reality and thus Hegel introduced dialectical method. Comment.
31. Hegel predominantly introduced the dialectical method to modern philosophy. Comment
32. "In the capitalist system, the human labour itself becomes a commodity. The human labour becomes a commodity with its own use-value and exchange-value. The capitalists buy, use, and sell this commodity just like other commodities." Explain the point from Marx's perspective.
33. "The individuals do not determine their needs themselves; it is the society that imposes the needs." Explain this statement from the perspective of the theory of hegemony in culture industry.

SECTION D

Long answer Question; Answer any three (10x3=30)

34. Analyze Kant's attempt for 'critique of reason'.
35. Analyze the core philosophical debate about the material production and mental production in Hegel and Marx's philosophy.
36. Do you support or not Marx's criticism of private property and capitalist economy? Critically evaluate and justify yourself.
37. Is thought (theory) a product of time and history? Or, is thought (theory) abstracted from time and history and thus universal? What is the take of Hegel, Marx and critical theorists in this debate? Apply your own reason in historical events and in your life for your argument and debate in detail.
38. "In a one-dimensional society, the powerful class use their elite ideology manipulatively to shape the aspirations, hopes, fears, and values of other individuals and thus this society is not characterized by unity and harmony; rather it is an embodiment of domination and oppression. Such a one-dimensional society produces one-dimensional humans who, being the object of domination, lack their individuality, autonomy, capacity to think, reflection and awareness of personal needs, creative self-activity, the ability to control one's own development, and the right to dissent." Do you see the relevance of Marcuse's statement in our current political scenario. Evaluate the statement with current instances.
39. Hegel's idealism, Marx's materialism and Critical theory, all can be seen as different versions of the criticism of enlightenment reason and philosophy. Analyze with your own justifications and views.

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - **M23PH06DC INDIAN PHILOSOPHY II (**CBCS - PG)

2023-24 - Admission Onwards

Time: 3 Hours Max Marks: 70

SECTION A

Objective Types Questions; Answer **any Ten** (1x 10=10)

1. Name the foundational text of the Nyāya school
2. Who is considered the author of the Brahma Sūtra?
3. In which stage of chitta are all mental modifications entirely stopped according to Yoga?
4. Name the main text of the Mimāṁsā philosophy authored by Jaimini?
5. Who is believed to be the author of the Vaiśeṣika Sutra?
6. What is considered as the material cause of the universe in Sāṅkhya philosophy?
7. Who is the founder of the Sāṅkhya system?
8. What is the primary focus of Nyāya philosophy
9. In Vedānta philosophy, the method 'Neti Neti' is used to describe Brahman as what?
10. Who is considered to have systematized the principles of Mimāṁsā?
11. What is the first evolute that emerges from Prakriti according to Sāṅkhya?
12. The Padarthadharmasangraha is authored by
13. What is the Tattva-traya according to Rāmānuja?
14. How many pramāṇas are accepted by Prabhakara in Mimāṁsā?
15. Who is the chief proponent of Viśiṣṭādvaita?

SECTION B

Very Short Answer Questions; answer **any Five (**2x5=10)

16. Define Brahmaparināmavāda.
17. How is Prāgbhāva understood in Vaiśeṣika philosophy?
18. What is apūrva?
19. Which are the four fundamental conditions for valid knowledge according to Nyāya?
20. What is chitta according to Yoga philosophy?
21. Which are the conditions that determine the validity of knowledge in Mimāṁsā philosophy?
22. Name the five kinds of afflictions or kleśas according to Yoga philosophy.
23. Name the pramāṇas accepted by Kumārila.
24. How is the concept of Jivan Mukti understood in Indian philosophy?
25. Explain the role of Puruṣa in Sāṅkhya philosophy.

SECTION C

Short Answer Questions; answer **any five (**4x5=20)

26. Evaluate the effectiveness of Madhva's epistemology in addressing the limitations of other Vedānta schools.
27. Analyse the concept of dharma in Mimāṁsā philosophy and its importance in guiding moral and social conduct.
28. Evaluate how well the three guṇas of prakṛti (sattva, rajas, tamas) explain the evolution of the universe in Sāṅkhya philosophy.
29. Explain how Mimāṁsā philosophy used arthāpatti as a pramāṇa to resolve inconsistencies in perceived facts.
30. Compare Sāṅkhya's Satkāryavāda with Vaiśeṣika's Asatkāryavāda with examples.
31. Describe the five stages of Chitta (Chitta Bhumi) and their characteristics in the context of Patanjali's Yoga Sūtras.
32. Discuss the significance of anupalabdhi (non-apprehension) in the philosophy of Kumārila Bhāṭṭa. How does it differ from perception and inference?
33. Critically evaluate the Vaiśeṣika theory of atomism in the context of contemporary scientific understanding.

SECTION D

Long answer Question; Answer **any three** (10x3=30)

34. Discuss the concept of 'qualified non-dualism' (Viśiṣṭādvaita) in the context of Rāmānuja's metaphysics. How does it differ from Shankara's Advaita Vedānta?

35. Assess the main teachings of Nimbārka's Dvaitādvaita. How does Nimbārka's interpretation of Vedānta differ from that of Vallabha?

36. Critically analyse Prabhākara's and Kumārila's theory of knowledge and error and its key components.

37. Examine the eight limbs of Aṣṭāṅga Yoga described by Patanjali in the Yoga Sutras. How do these eight states collectively contribute to the holistic development of an individual, both physically and spiritually?

38. Explain the concept of inference (anumāna) in Nyāya philosophy and describe how it works. What are the different types of inference recognised in Nyāya, and how do Hetvābhāsas affect this process?

39. Discuss the seven fundamental categories (padārthas) in Vaiśeṣika philosophy. How do they contribute to understanding the nature of reality?

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - **M23PH06DC INDIAN PHILOSOPHY II** (CBCS - PG)

2023-24 - Admission Onwards

Time: 3 Hours Max Marks: 70

SECTION A

Objective Types Questions; Answer any Ten (1x 10=10)

1. Who is considered as the founder of Nyāya philosophy?
2. Name the two major schools of Mīmāṃsā philosophy
3. What are the two fundamental realities in Sāṅkhya dualism?
4. What is the main focus of Pūrva Mīmāṃsā philosophy?
5. Name the eight limbs of Aṣṭāṅga Yoga in their correct order
6. In Vallabha's philosophy, the concept of "Pushti" refers to
7. Define Purusha according to Sāṅkhya philosophy.
8. What is the primary focus of Vaiśeṣika philosophy compared to Nyāya?
9. In Advaita Vedānta, which concept represents the highest level of reality?
10. In Nyāya philosophy, what does Pramā refer to?
11. How many pramāṇas are accepted by the Vaiśeṣika school of philosophy?
12. According to Purva Mīmāṃsā, the term 'apūrva' refers to
13. Who is considered as the founder of the Viśiṣṭādvaita school of Vedānta?
14. Name the Pramāṇas recognized by Jaimini.
15. What does the concept of Āvirbhāva refer to in Vallabha's philosophy?

SECTION B

Very Short Answer Questions; answer any Five (2x5=10)

16. Name the pramāṇas accepted by Prabhākara
17. Define 'Dravya' in Vaiśeṣika philosophy.
18. What is the goal of Aṣṭāṅga Yoga?
19. Define Sabda in Nyāya epistemology.
20. What are the three types of bhaktis accepted by Rāmānuja
21. Name major characteristics of Prakṛti in Sāṅkhya philosophy
22. Describe the two types of Jīva accepted by Nimbārka
23. Describe the concept of Upamāna in Nyāya epistemology.
24. Describe the three-member syllogism accepted by Mimāṁsā philosophy
25. Explain the concept of Apara Brahman in Advaita Vedānta.

SECTION C

Short Answer Questions; answer any five (4x5=20)

26. Explain 'Abhāva' and its kinds in the Vaiśeṣika system of metaphysics.
27. How Mimāṁsā scholars interpret Dharma as the guiding principle for ethical behavior and ritual practice, and analyse the role of Apurva in connecting actions to their unseen results.
28. Elaborate on the Sāṅkhya theory of evolution
29. Critically evaluate the relationship between Atman and Brahman according to Advaita Vedānta?
30. Explain the concept of the three levels of reality in Śaṅkara's Advaita Vedānta. How do the distinctions between them help in understanding the nature of existence?
31. Analyse the importance of grace and bhakti in Vallabha's philosophy
32. Explain how Hetvābhāsa affects the validity of an inference in Nyāya and provide examples of common fallacies.
33. Analyse the distinction between valid and invalid knowledge according to Nyāya philosophy.

Section D

Long answer Question; Answer any three (10x3=30)

34. Compare and contrast metaphysical conceptions of Rāmānuja's Viśiṣṭādvaita with Madhva's Dvaita.

35. Discuss how Pūrva Mimāṁsā scholars define and classify different pramāṇas in their philosophy. Analyse the emphasis placed on śabda pramāṇa within this school and compare it with its treatment in Nyāya thought.

36. Discuss in detail Prabhākara's and Kumārila's theories of knowledge and error. Explain the fundamental components of each theory, compare their approaches, and evaluate the philosophical implications of their differences.

37. Analyse the Nyāya theory of Perception (Pratyakṣa). Discuss its definition, types, and the process by which perception leads to valid knowledge.

38. Analyse the concept of dualism in Sāṅkhya philosophy. Discuss the relationship between Purusha (consciousness) and Prakriti (matter), highlighting their distinct characteristics and interactions.

39. Examine the Theory of Evolution in Sāṅkhya philosophy. Discuss the different stages of evolutionary process in the Sāṅkhya system.

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - **M23PH07DC: SYMBOLIC LOGIC** (CBCS - PG)
2023-24 - Admission Onwards

Time: 3 Hours | Max Marks: 70

SECTION A

Objective type questions. Answer any 10 **10x1=10**

1. ______ Logic focuses on the truth values of propositions rather than their content.
2. ____________ method involves deducing particular conclusions from general premises.
3. Write the name of the proposition: "Some roses are red."
4. How do we Symbolize: "Krishna is a student and Parvathi is intelligent"?
5. What is proposition in Logic?
6. Name the statement form that cannot be true in any of its substitution instances.
7. Write the symbolization for "either ... or."
8. If we are drawing a truth table with 5 components, how many rows will we have to draw?
9. Write the name of the argument given below:

   $p \supset q$

   $\sim q$

   $\therefore$ p

10. Name the double negation in the rule of replacement in Logic?
11. Write the name of the technique that can quickly identify an invalid argument without evaluating all possibilities.
12. According to predicate logic, how do we Symbolize the proposition "Krishnapriya is a scientist".
13. How do we symbolize 'A' proposition in quantification

14. How to symbolize existential instantiation in Logic?

15. What is a singular proposition in predicate logic?

SECTION-B

Very Short answers questions. Answer **any 5** 5x2=10

16. What is inclusive disjunction?

17. What is an argument form in Logic?

18. Define the rule of distribution in the rule of replacement.

19. Provide a shot note on indirect proof

20. How is a general proposition defined in Logic?

21. Define the 'I' proposition in predicate logic.

22. Define the methods of quantification.

23. Write any two advantages of symbolic logic.

24. Define a variable.

25. What is the law of the excluded middle in Logic?

SECTION C

Short Answer Question answer **any 5** 5x4=20

26. Explain how symbolic logic differs from traditional logic.
27. Describe the relationship between truth and validity.
28. "The student is either intelligent or not intelligent"—explain the notions of truth-functional logic.
29. Describe the truth table of biconditional or logical equivalence and draw its truth table.
30. Explain the rule of inference by providing any two rules.
31. Compare general and singular propositions with examples.
32. Describe the Square of Opposition in predicate logic.
33. Compare the relationship between universal and existential quantification.

SECTION D

Long Answer questions answer **any 3** 3x10=30

34. Describe simple and compound statements. Discuss different types of compound statements with their truth table

35. Explain the rules of replacement and their applications.

36. Compare conditional proof and indirect proof.

37. Discuss how quantification differs from propositional logic. Distinguish between universal quantifiers (∀) and existential quantifiers (∃).

38. Explain the shorter truth table method. How does it prove the argument given below.

    (~ p . q). (q ͻ P)

39. Discuss statement forms and explain the different types of statement forms.

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - **M23PH07DC: SYMBOLIC LOGIC** (CBCS - PG)

2023-24 - Admission Onwards

Time: 3 Hours Max Marks: 70

SECTION A

Objective type questions. Answer any 10 **10x1=10**

1. A categorical proposition possesses both ________ and ________.
2. Name the proposition: "All teachers are lawyers."
3. In logic, the statements that are either true or false are called ________.
4. In symbolic logic, write the symbols for conjunction and inclusive disjunction
5. How do we Symbolize the proposition "Trivandrum is the capital of Kerala"
6. _______ statements contain simple statements as their components.
7. What is a specific form?
8. How do we symbolize the phrase "if and only if"?
9. Write the name of the argument. $p \supset q$

   p

   ∴ q
10. What is meant by tautology in the rule of replacement?
11. The method of indirect proof is also called the method of proof by __________.
12. What is the method of deduction that demonstrates the validity of an argument by assuming the antecedent of a conditional statement?
13. Write an individual variable in predicate logic
14. Write the 'O' proposition in quantification
15. What are general propositions in predicate logic?

SECTION-B

Very Short answers questions. Answer **any 5** 5x2=10

16. Define validity of an argument.
17. How is the law of identity used in logical reasoning?
18. What is meant by a constant in logical term, and how is it used in logical expressions?
19. Define a tautology in propositional logic.
20. Define contingent proposition.
21. Define Modus Ponens
22. Explain the rule of Association in rules of inference.
23. Explain singular propositions in predicate logic with an example.
24. Discuss the E proposition in predicate logic.
25. How does negation affect the truth value of a proposition?

Section C

Short Answer Question. Answer **any 5** 5x4=20

26. Discuss propositional logic.
27. How important are brackets in logic? Explain with examples.
28. Differentiate between variables and constants.
29. Explain conditional proof with an example.
30. Explain the rules of inference with any two specific rules.
31. Discuss the shorter truth table method for determining the validity of arguments
32. Compare general and singular propositions with examples.
33. Explain the Square of Opposition in predicate logic with its structure.

SECTION D

Long Answer questions answer **any 3** 3x10=30

34. Describe the relationship between universal and existential quantification.

35. Discuss the difference between propositional logic and predicate logic. Explain the technique of symbolization in propositional logic

36. Elucidate the differences between classical and symbolic logic. Explain the advantages of symbolic logic and the limitations of classical logic.

37. Describe the truth table method for proving the validity of an argument. Analyze the specific argument form given below using a truth table.

$p \supset q$

P

$\therefore q$

38. Compare simple and compound statements. Discuss the different types of compound statements.

39. Explain the Preliminary Quantification Rule and its significance in predicate logic. Prove the validity arguments by using the same.

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION

DISCIPLINE CORE - M23PH08DC SOCIO-POLITICAL PHILOSOPHY (CBCS - PG)

2023-24 - Admission Onwards

Time: 3 Hours Max Marks: 70

SECTION A

Objective Types Questions; Answer **any Ten** (1x10=10)

1. Who authored *The Republic*?
2. What is the term used by Aristotle for the ideal state?
3. In Hobbesian theory, what is the term for the pre-state condition?
4. Which philosopher is known for the concept of "veil of ignorance"?
5. What is the name of Rawls' concept that addresses justice?
6. Who wrote *The Origins of Totalitarianism*?
7. What is the term used by Berlin to describe freedom from interference?
8. Which philosopher is known for the theory of social justice in 20th century India?
9. What is the term for the agreement to form a political society according to Rousseau?
10. Which thinker is associated with the theory of class struggle and revolution?
11. What is the term for the principle of not interfering in individual choices according to Nozick?
12. What is the term for the philosophical study of politics?
13. Who wrote *Two Treatises on Government*?
14. What concept does Amartya Sen critique in relation to justice?
15. What is the term for the principle of non-interference in one's life according to libertarians?

SECTION B

Very Short Answer Questions; Answer **any Five** (2x5=10)

16. What is the primary focus of political philosophy?
17. What is the main idea of Plato's *Republic*?
18. How does Aristotle define citizenship?
19. What is the core idea of Hobbes' Leviathan?
20. Summarize Rousseau's concept of the social contract.
21. Describe Rawls' idea of veil of ignorance.
22. What does Berlin mean by positive liberty?
23. Define the term social justice as proposed by Ambedkar.
24. Explain the concept of totalitarianism according to Arendt.
25. What is the main argument of Nozick's libertarian theory?

SECTION C

Short Answer Questions; Answer **any Five** (4x5=20)

26. Explain Plato's idea of justice as presented in *The Republic*.
27. Compare Aristotle's and Plato's views on the role of the state in society.
28. Discuss Hobbes' view on the state of nature and its implications for political theory.
29. Summarize John Rawls' theory of justice as fairness.
30. Explain Amartya Sen's critique of Rawlsian justice.
31. Describe Robert Nozick's concept of the minimal state and its justification.
32. Discuss Hannah Arendt's views on the nature of totalitarianism.
33. Compare and contrast Isaiah Berlin's concept of negative liberty with Charles Taylor's critique.

SECTION D

Long Answer Questions; Answer **any Three** (10x3=30)

34. Evaluate Karl Marx's theory of class struggle and its impact on contemporary political thought.
35. Discuss the nature and scope of political philosophy.

36. Compare and contrast the key principles of Hobbesian and Lockian social contract theories. Discuss the strengths and weaknesses of each theory.

37. Examine the theories of justice proposed by John Rawls and Amartya Sen. How do their views differ?

38. Analyse the debate on secularism in India as discussed by Romila Thapar and Irfan Habib.

39. Discuss the significance of inclusion and exclusion in contemporary political theory. How do multiculturalism and identities play a role?

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SECOND SEMESTER MA PHILOSOPHY EXAMINATION
DISCIPLINE CORE - M23PH08DC SOCIO-POLITICAL PHILOSOPHY (CBCS - PG)

2023-24 - Admission Onwards

Time: 3 Hours Max Marks: 70

SECTION B

Objective Type Questions; Answer ***any Ten*** *(1x10=10)*

1. Who wrote *The Politics*?
2. What is the term used by Plato for the highest form of knowledge?
3. In Locke's theory, what is the term for natural rights that exist before the establishment of government?
4. Which philosopher is associated with the concept of general will?
5. What is the title of Amartya Sen's book that critiques Rawlsian justice?
6. Who is the author of *On Liberty*?
7. What term does Nozick use to describe a state that only protects against force, theft, and fraud?
8. Which philosopher introduced the idea of positive liberty?
9. What is the term for the collective agreement to create a society as per Hobbes?
10. Which thinker is associated with the concept of revolution in relation to class struggle?
11. What is the philosophical term for a society governed by social rules as per Rousseau?
12. Who coined the phrase Two Concepts of Liberty?
13. What is the term for Aristotle's classification of constitutions?
14. Who developed the theory of Public Reason?
15. What is the term for the political philosophy that emphasizes individual freedom from state control?

SECTION B

Very Short Answer Questions; Answer ***any Five*** *(2x5=10)*

16. What is Aristotle's idea of a good life?
17. How does Plato define the concept of the philosopher-king?
18. Summarize the main argument of Locke's *Two Treatises on Government*.
19. Describe Rousseau's view on the state of nature.
20. What is the main idea behind Amartya Sen's capability approach?
21. Explain the concept of positive liberty as discussed by Berlin.
22. Define Robert Nozick's entitlement theory.
23. What is the significance of the notion of veil of ignorance in Rawls' theory?
24. Describe Ambedkar's perspective on social justice.
25. What are the key features of totalitarianism according to Hannah Arendt?

SECTION C

Short Answer Questions; Answer ***any Five*** *(4x5=20)*

26. Discuss the importance of the idea of justice in Plato's *Republic*.
27. Compare and contrast the political theories of Hobbes and Rousseau.
28. Explain the role of the state according to Aristotle in *The Politics*.
29. Summarize Isaiah Berlin's distinction between negative and positive liberty.
30. Discuss Karl Marx's view on historical materialism.
31. Analyse John Rawls' concept of "justice as fairness" and its relevance today.
32. Explain the differences between the social contract theories of Hobbes and Locke.
33. Discuss the implications of totalitarianism as described by Hannah Arendt.

SECTION D

Long Answer Questions; Answer ***any Three*** *(10x3=30)*

34. Critically evaluate the theories of social contract as proposed by Locke and Rousseau. How do they differ in their views on human nature and the justification of the state?

35. Evaluate the relevance of Ambedkar's theory of social justice in the context of contemporary Indian society.

36. Analyse the contributions of Aristotle to political philosophy and how his ideas differ from those of Plato.

37. Discuss the evolution of the concept of justice from classical to contemporary political philosophy, with reference to the works of John Rawls and Amartya Sen.

38. Examine the philosophical debates on freedom in contemporary political theory, focusing on the works of Isaiah Berlin and Charles Taylor.

39. Analyse the contemporary debates on secularism in India, with special reference to the works of Prabhat Patnaik and Rajeev Bhargava.

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Symbolic Logic

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